The New Mathematics for Primary Teachers

7. More about Shapes

D. Paling and J. L. Fox

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THE NEW MATHEMATICS
FOR PRIMARY TEACHERS

7. More about Shapes

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Preface to the series

At the present time there is a growing dissatisfaction with mathematical work in schools. There is a feeling that the content of courses does not fit in with the present-day needs of science and technology, industry and trade. At the same time the attitude of teachers towards the teaching of mathematics is undergoing a fundamental change. More and more teachers are coming to feel that teaching by 'rule' is not good enough. They want children to find out more for themselves, to question what they are doing and find their own ways of solving problems. They want children to understand rather than to learn like parrots.

These two movements–the modernisation of the content of mathematics and the shift of emphasis from 'the teacher teaching' to 'the child learning'–are going on together in many countries. They both make large demands on teachers, especially in the primary school. These teachers now need to take a new look at mathematics itself and at the same time try to introduce more enlightened methods in the classroom.

If these changes are to come about, teachers in schools and students in training will need all the help they can be given. It is for this purpose that this series of short books has been written. The aim is to give teachers and student-teachers a new mathematical background so that they can introduce modern mathematics in the primary school. The books do not set out to deal with the classroom situation itself but they do present, at teacher level, the kind of approach now being introduced into schools. The completed set of books provides teachers with a mathematical background for their work in the primary school. The approach is simple and straightforward and no previous knowledge is assumed. Many exercises (with answers) are provided at each stage; they need to be worked through very carefully. The exercises not only consolidate the work already done but provide the starting-point for the next stage.

St Luke's College, Exeter
March, 1973

D.P.
J.L.F.

CONTENTS

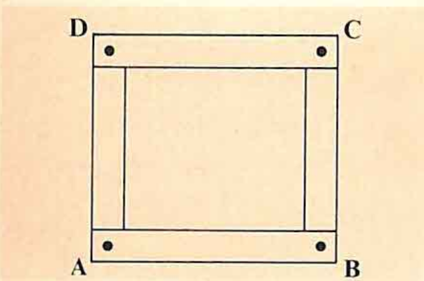
1. INTRODUCTION

From the earliest of times man has been interested in the shapes of the many objects which he sees around him. Some of these objects are produced by nature—an orange, a leaf, a pebble, a mountain, the moon, a shell. Others are man made—a table, a cup, a hoe, a pyramid, a canoe, a concrete block. There is an infinite set of objects and an infinite set of shapes.

Over the years man has built up a store of knowledge about the more common shapes and has used this knowledge in building and in technology. He has also devised ways of comparing the sizes of shapes through measuring length, area, volume, angle, etc.

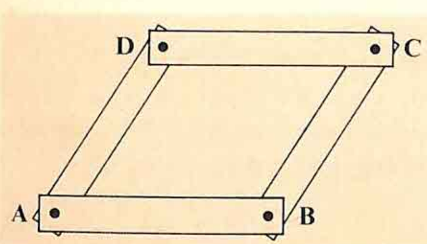
Man quickly came to learn that the shape of some objects can be changed. For example: a ball of clay can be changed to a cube; a spring can be stretched or compressed; a piece of elastic can be stretched lengthways or side ways. So not only do we want to know about the shapes of objects at rest but also we need to know about the ways in which shapes change.

A simple piece of apparatus brings out another aspect of what we need to consider in studying shapes.



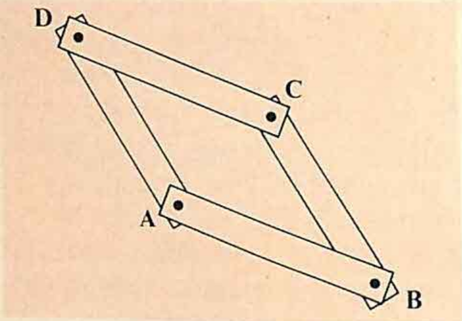
The drawing shows four strips of cardboard joined together by paper fasteners to make the outline of a square. The corners are labelled A, B, C and D.

Now consider some of the ways in which the shape can be changed. Here is one way:



A and B are held fixed whilst C and D are moved to the right.

Here is another way:



A is held fixed whilst C is moved towards A.

If we compare each of the two new shapes with the original square we see that:

(a) the shape has changed; the angles have changed; the area has changed

(b) the lengths of the edges are unchanged; the number of edges is unchanged; the opposite edges are still parallel: the number of angles is unchanged; the opposite angles are still equal.

Of (a) and (b) it is the latter which is the more important. It enables us to say that any rhombus made from the square will have its opposite edges parallel and its opposite angles equal.

Sometimes we need to make copies of a shape. Very often the copy is exactly the same as the original shape and the two shapes are said to be congruent.

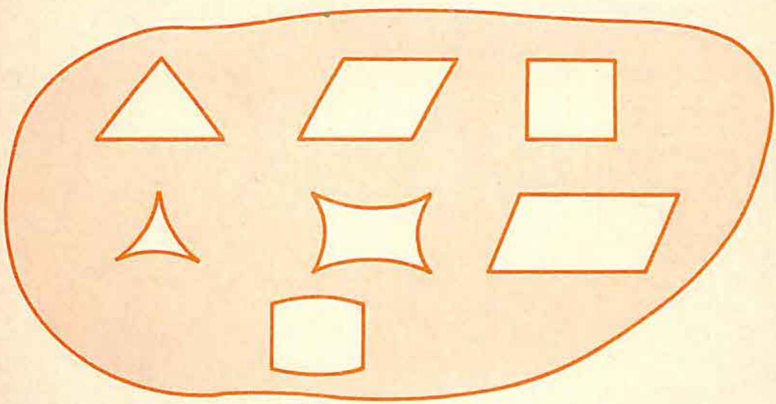
Sometimes the copy is a reduced version of the original (as in a plan of a building or in a map) and sometimes it is an enlarged version (as a small insect may appear under a microscope).

In all these cases the two shapes are said to be similar. We need to know about congruency and similarity if we are to make full use of our knowledge of shapes.

We also need to be able to recognise that some shapes are symmetrical in one way or another. This often enables us to decide whether a particular part of a shape is congruent with another part. When this is so we are able to say that certain faces, edges or angles are equal to each other.

When we were young we all started to build up ideas about the shapes of the objects we saw around us. We began to recognise, for example, the kind of shapes used as containers. We began to see how the shapes of a brick, a concrete block and a case were alike. We became aware that some surfaces were flat whilst others were curved. We did not use mathematical terms to describe our ideas but we were all the time building up a store of knowledge about shapes. We were beginning to sort the shapes out in our minds into those that were alike in some way and those that were different.

This kind of thinking is a good foundation for our further study of shapes, for much of what we do as we progress stage by stage is concerned with sorting shapes according to their various properties. For example, if we wished to investigate the properties of the shapes in the set shown below, we would probably separate them into two subsets.



Either

(a) shapes with straight edges and (b) shapes with curved edges;
or (a) shapes with three edges and (b) shapes with four edges.

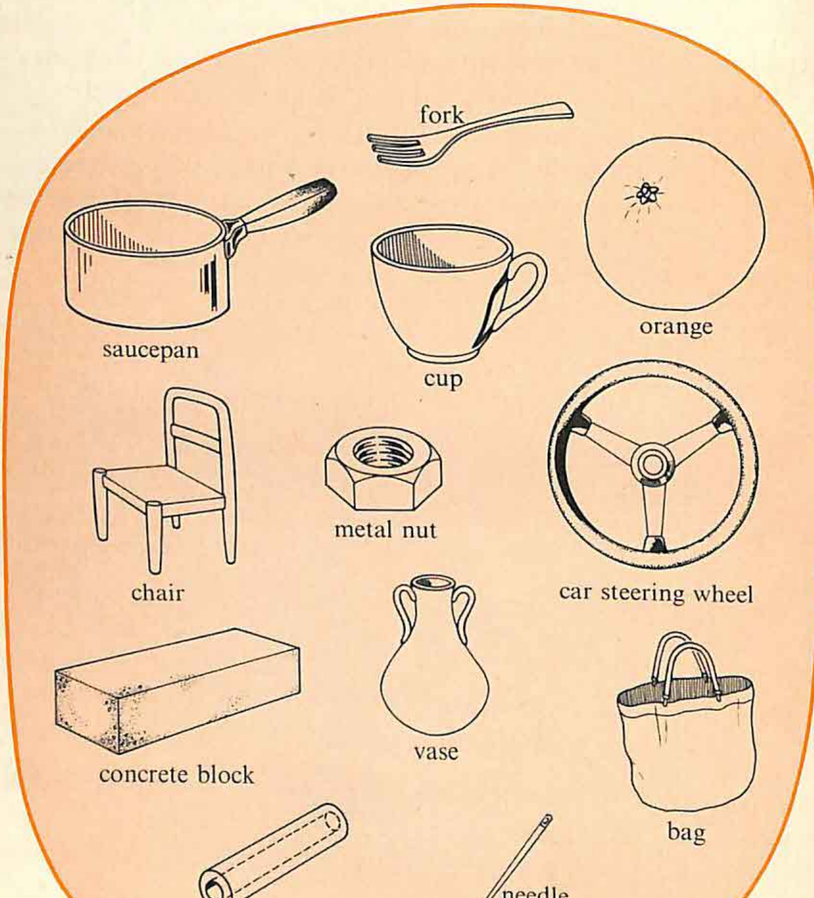
We shall see how this kind of sorting develops as we work through the book.

Important ideas in this chapter

1. The objectives in a study of shapes.
2. An approach to a systematic study of shapes.

2. EQUIVALENT SHAPES

A. How many holes?



Exercise

1. Look at the set of objects shown on page 4.
   In what ways are they alike? In what ways do they differ?
   Think of several different ways in which they could be sorted.
   Show some of your sortings by drawings.

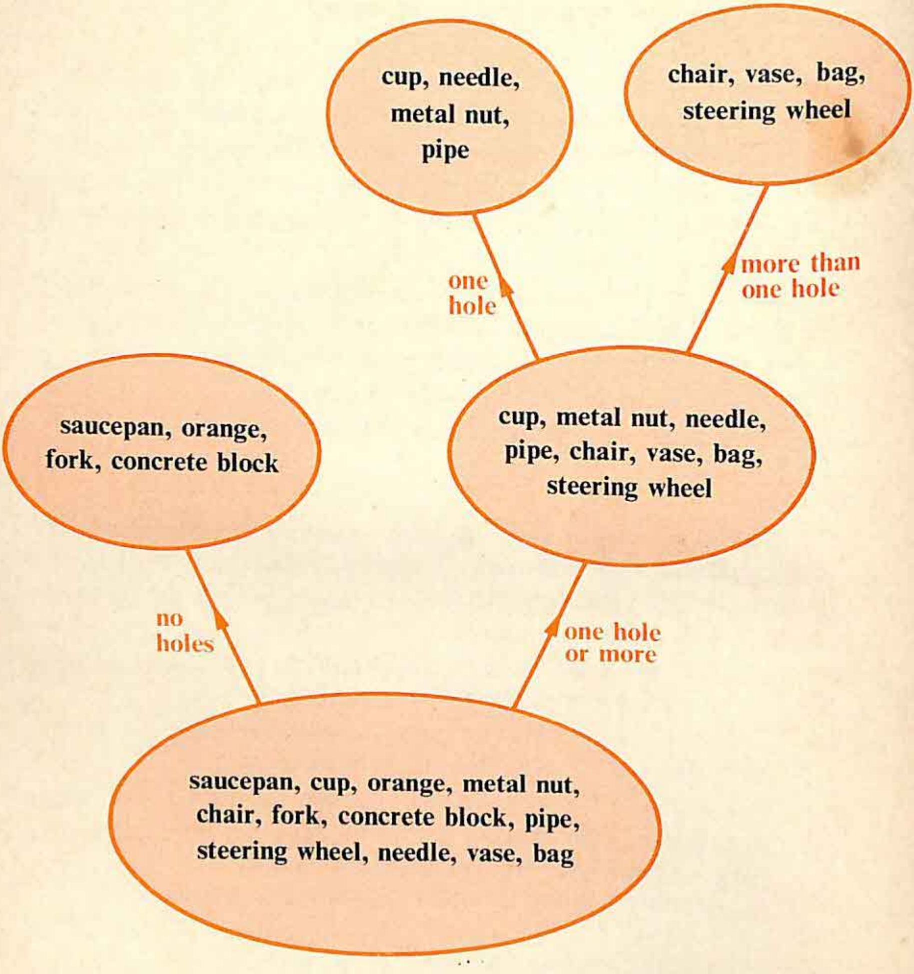
There are, of course, many ways of sorting the set of objects shown on page 4. The sorting can be done by the material of which each object is made, by its colour, by its size, by the use to which it is put, by its shape, and so on. We could use any of these sortings, but for the time being we are going to sort the objects by considering their shapes.

When we first look at the set of objects it does not seem easy to do this sorting: the shapes of the objects seem so different. However there may be some similarities. We shall try to find these through activities using models of the shapes. These models can be made from clay or other available material.

Exercise

2. Use a lump of clay or other suitable material to make a model of each of the objects shown on page 4. These models need not be to scale, nor need they be very accurate, provided that they generally resemble the objects.
   The lump of clay used for a model should not be cut or broken into pieces during the making of the model.
   (a) Which of the models can be made without making a hole? (N.B. Do not confuse hole with hollow.)
   (b) For which of the models is it necessary to make one hole?
   (c) For which of the models are more than one hole needed?
   (d) Do you agree with the statements below?
      (i) Models needing no holes: {saucepan, orange, fork, concrete block}
      (ii) Models needing one hole: {cup, metal nut, needle, pipe}
      (iii) Models needing more than one hole: {chair, car steering wheel, vase, bag}

The sorting which leads to the statements in Exercise 2(d) can be shown in several ways. Here is one way.



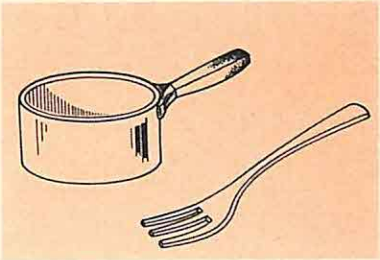
This kind of diagram is often called a **sorting tree**. It is a simple but very helpful way of planning, as well as showing, useful sortings. As much of our work in this book will be based on sorting shapes, we shall make frequent use of sorting trees.

B. Topological equivalence

Exercises

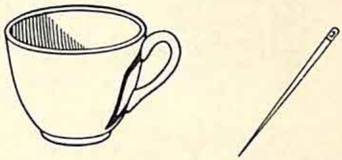
3. Using clay or other available material, make a model of a saucepan.

   Without breaking, cutting or making a hole through the clay, can you remould it into the shape of the fork?



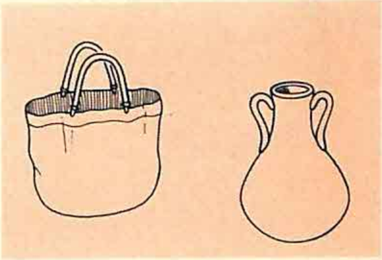
4. Using clay or other material make a model of a cup.

   Keeping the hole formed by the handle (and without cutting, breaking or making other holes) can you change the model of the cup into a model of the needle?



5. Using clay or other material make a model of the bag.

   Keeping the holes formed by the handles can you change the model of the bag into a model of the vase?

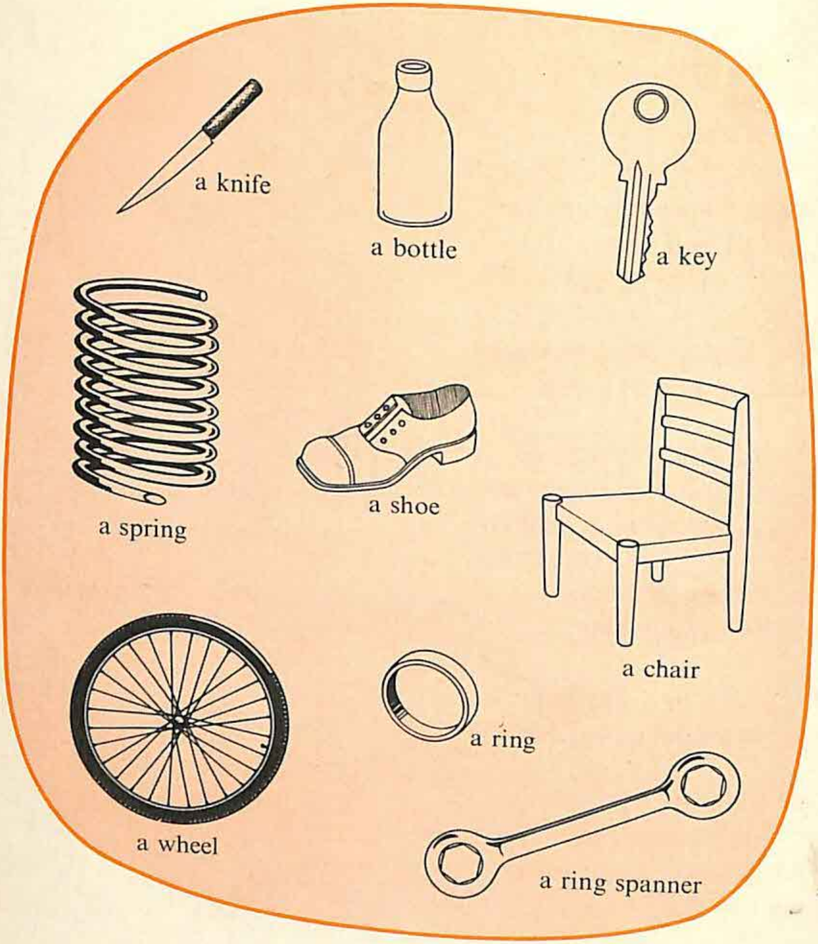


When a clay model of an object can be changed into a model of another object (keeping the rules used in Exercises 3 to 5 above) we say that the two objects have shapes which are **topologically equivalent**.

6. Make a collection of objects whose shapes are topologically equivalent to:
   (a) a pin (b) a gramophone record.

EQUIVALENT SHAPES

7. Copy the outline of the sorting tree on page 6. Use the tree to sort the objects shown below.



Which of the objects have shapes which are topologically equivalent to: (a) the knife? (b) the key? (c) the shoe?

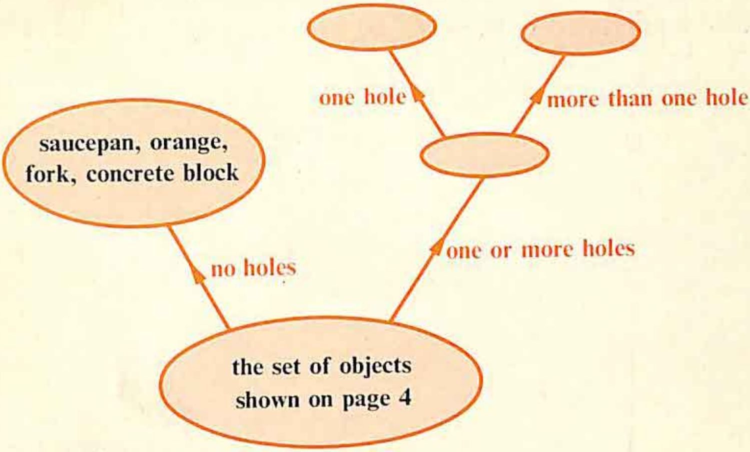
Important ideas in this chapter

1. The idea of topological equivalence.
2. The idea of a sorting tree.

3. SORTING 3-D SHAPES

A. Sorting objects with no holes in them

In Chapter 2 our sorting of objects was shown by a tree, as below

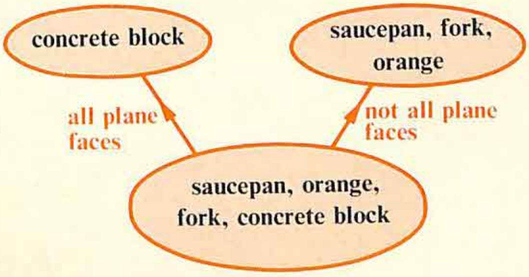


We are now going to consider a further sorting of the objects with no holes in them.

Exercise

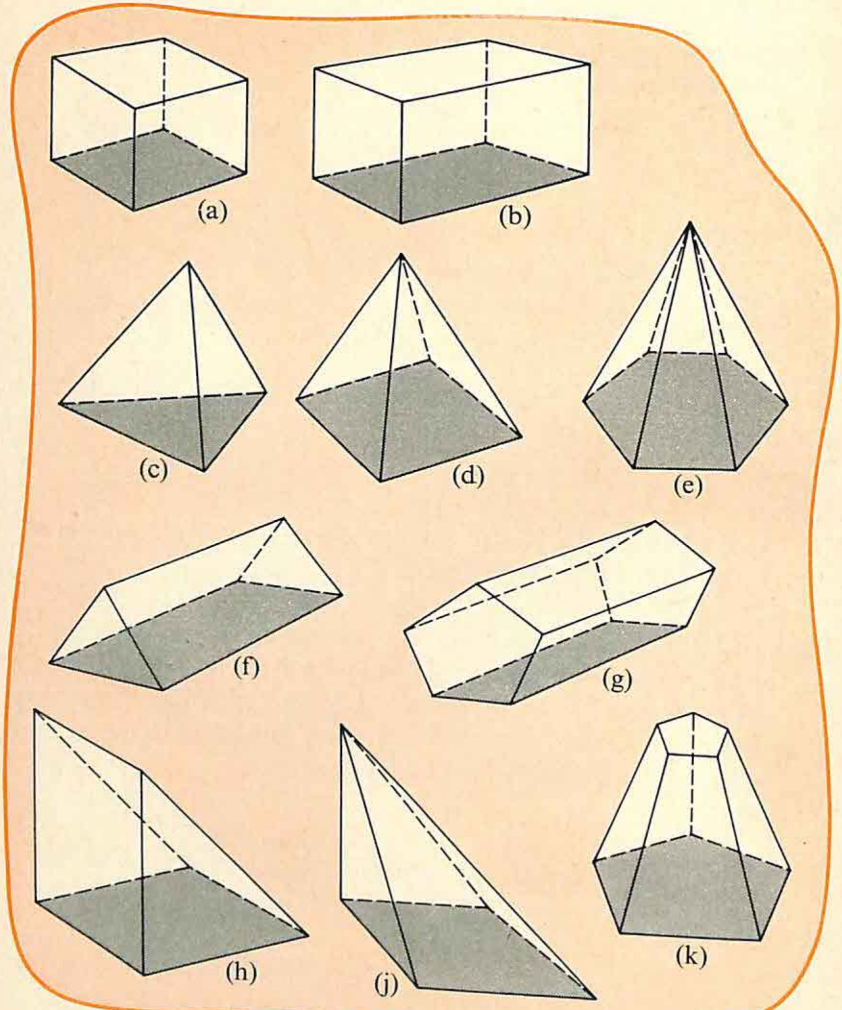
1. Suggest ways in which {saucepan, orange, fork, concrete block} could be sorted into two subsets.

This sorting can be done quickly and simply by considering the kind of faces each object has. All the faces of the block are planes. The orange has a curved face. The saucepan has some faces plane and some curved. The fork also has a mixture of curved and plane faces. The sorting can be shown as:



B. Shapes with all plane faces

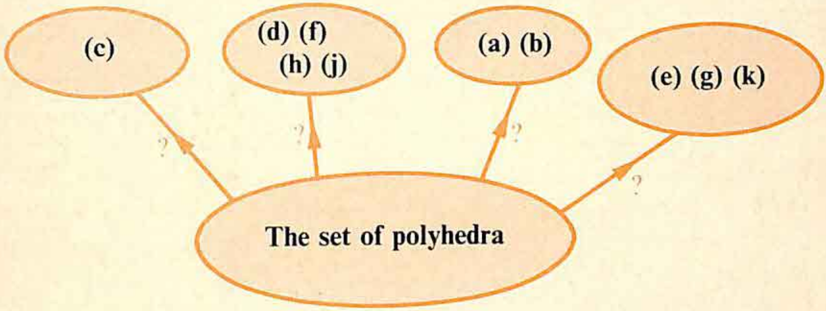
There is, of course, an infinite set of objects with all their faces plane. We will restrict ourselves to those which have common shapes. Also, for simplicity, we will consider the shapes rather than the objects themselves. A shape with all its faces plane is called a **polyhedron**. Here is a set of polyhedra:



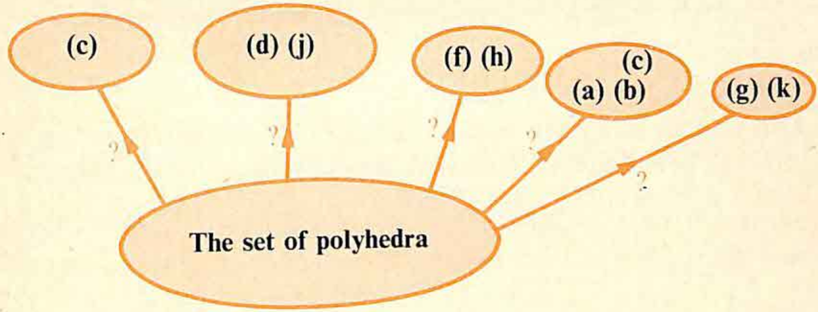
Exercises

2. Sort the shapes shown on page 10 in several different ways.

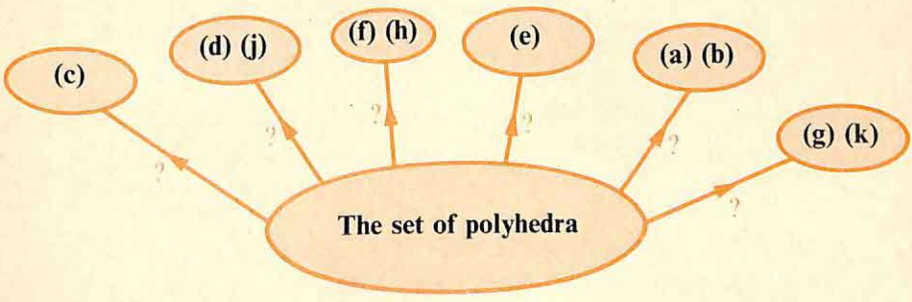
3. Label each arrowed line in this sorting (based on the number of faces) of the polyhedra shown on page 10.



4. Label each arrowed line in this sorting (based on the number of edges) of the polyhedra.



5. Label each arrowed line in this sorting (based on the number of vertices) of the polyhedra.



C. Vertices, edges and faces

In Exercise 3 the polyhedra are sorted according to the number of faces. In Exercise 4, the number of edges is used, whilst in Exercise 5 the number of vertices decides the sorting.

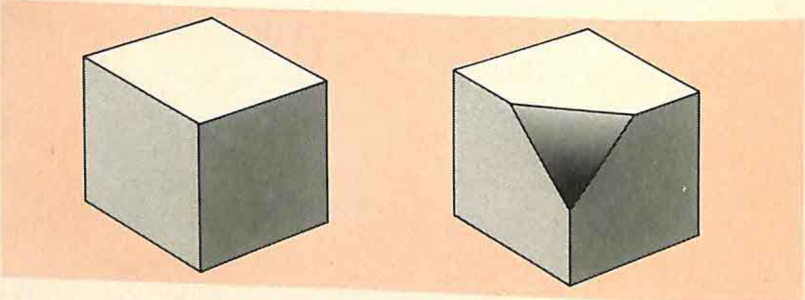
The information used in these exercises can be shown in a table, as below. The letters refer to the shapes on page 10.

Shape	Number of vertices	Number of faces	Number of edges
a	8	6	12
b			
c			
d			
e			
f			
g			
h			
i			
j			
k			

Exercises

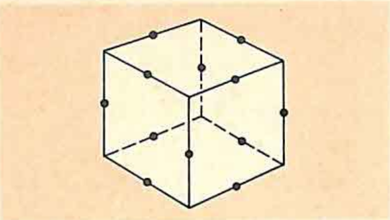
6. Complete the table above.
   Can you see any relationship between the number of vertices, the number of faces and the number of edges for each shape?

7. Writing **V** for the number of vertices, **F** for the number of faces, and **E** for the number of edges, is it true for each shape that
   $$\mathbf{V} + \mathbf{F} = \mathbf{E} + 2\ ?$$

8. 

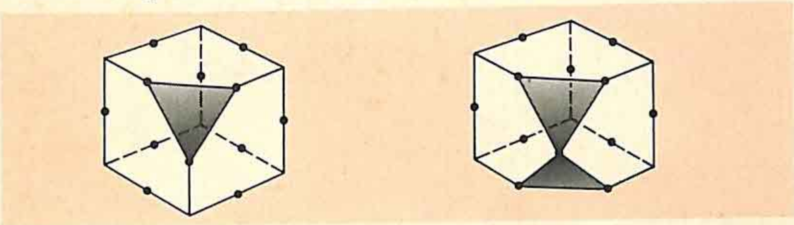
   Look at the cube. Check that $\mathbf{V} + \mathbf{F} = \mathbf{E} + 2$.
   Look at the cube with a corner removed.
   Is it still true that $\mathbf{V} + \mathbf{F} = \mathbf{E} + 2$?

9.



Here is a cube with the mid-point of each edge marked.

One by one the corners of the cube are removed.
The drawings below show the first two stages.



Make drawings to show each of the other stages.
Or, even better, make a cube from clay, a yam or any other suitable material, and cut off the corners in turn.
At each stage, count the vertices, edges and faces and check to see whether it is true that $\mathbf{V} + \mathbf{F} = \mathbf{E} + 2$.

10. Use drinking straws, joined together with pieces of bamboo shoot or other suitable material, to make an outline model of the final shape you obtained in Exercise 9.

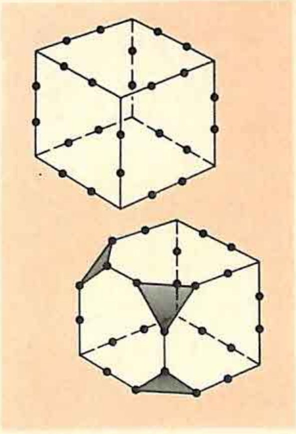
11. Here is a cube with points marked to divide each edge into three equal parts.

In turn, corners are removed as shown.

By making a model or by a set of drawings show each stage.

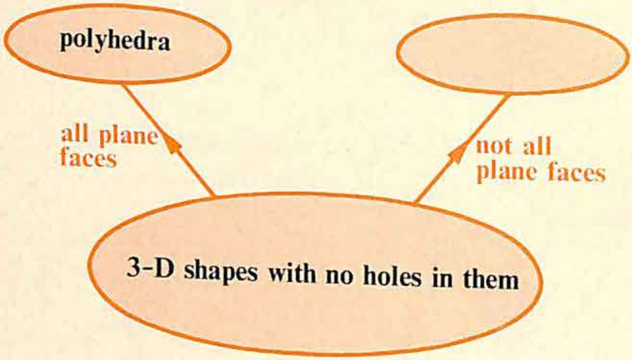
Check each time that $\mathbf{V} + \mathbf{F} = \mathbf{E} + 2$.

Make an outline model of the final shape.

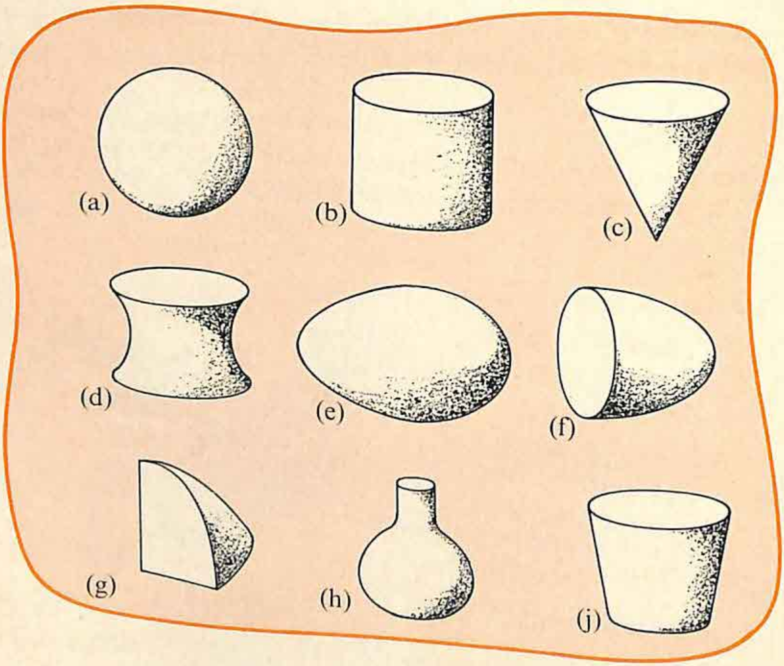


D. Shapes with not all their faces plane

In our sorting of 3-D shapes with no holes in them we have so far used:



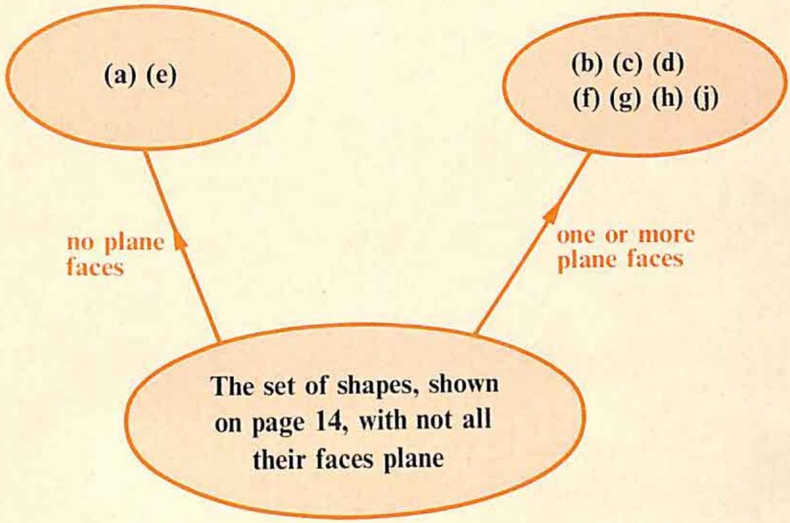
We now investigate some of the shapes which do not have all their faces plane.
Here are some of them:



Exercise

12. Decide upon ways of sorting the set of shapes shown on the opposite page. Use a tree to show each sorting.

There are, of course, many ways of sorting the set of shapes. Here is one of them:



In everyday life there is, of course, a very large set of objects with no plane faces. We have only to think of pebbles, beans, seeds, etc., to realise how many such objects there are.
The shapes which we most easily recognise among those with some curved faces are the sphere, the cylinder and the cone.

Important ideas in this chapter

1. The set of shapes with all plane faces (polyhedra).
2. The relationship between the number of faces, the number of edges and the number of vertices of a polyhedron.
3. The sorting of shapes with some curved faces.

4. SORTING 2-D SHAPES

A. Open and closed shapes

Here are some of the infinite set of plane shapes which could be made with a piece of string:

Exercise

1. Decide upon ways of sorting this set of shapes. Show your results on sorting trees.

There are many ways of sorting the set of shapes shown on page 16. A simple way of starting the sorting is shown below.

Exercise

2. Suggest a possible phrase to write by each of the arrowed lines in the above sorting tree.

The shapes in the top left-hand set are called **closed** shapes.
The shapes in the right-hand set are called **open** shapes.

Exercise

3. Draw another set of: (a) closed shapes (b) open shapes.

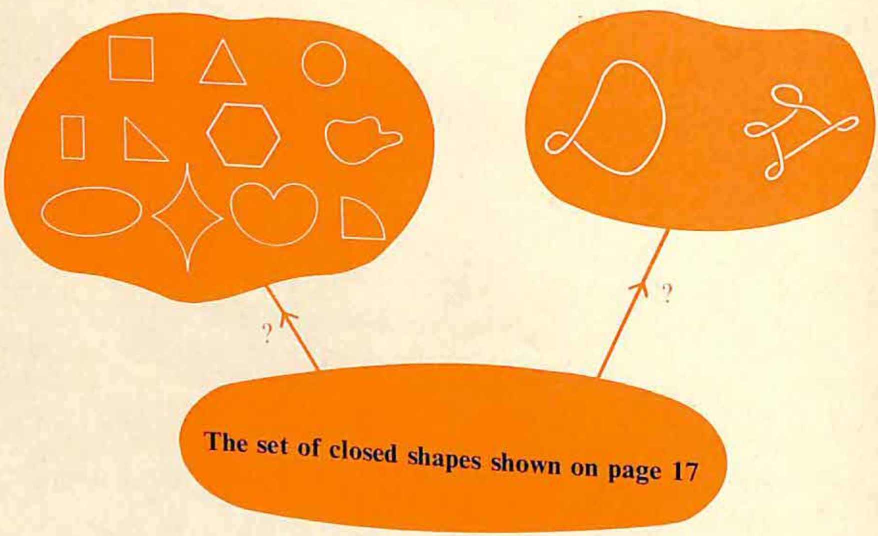
SORTING 2-D SHAPES

In everyday life we see and handle more closed shapes than open shapes. This does not mean that open shapes are less important than closed shapes. It does mean, however, that we are more familiar with closed shapes. This influences us to find out more about the closed shapes before we go on to investigate the open shapes. To do this it will be helpful to make further sortings of our set of closed shapes.

Exercise

4. Suggest ways of sorting the set of closed shapes shown on page 17. Use a sorting tree to show each of your ways.

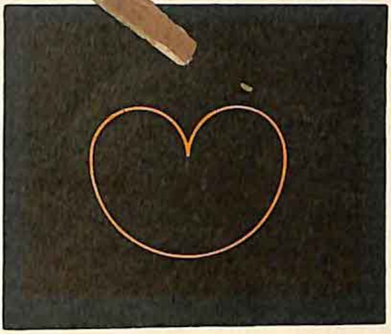
Again, there are many ways of doing this sorting. The sorting shown below, however, has the advantage of separating the simple shapes from the rather more complicated shapes.



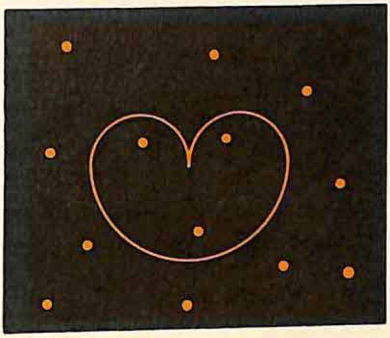
Exercise

5. For each of the arrowed lines in the above sorting tree suggest a possible phrase to write by it.

One way of finding a possible answer to Exercise 5 is to draw each shape in turn on a blackboard. For example:

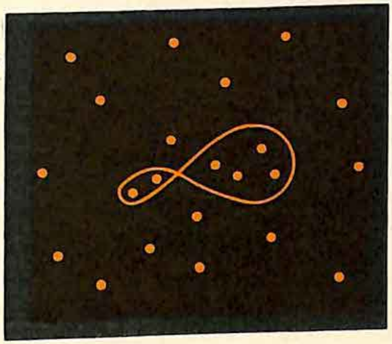


If we mark any point on the blackboard, apart from points on the shape itself, then it must be either inside or outside the shape.
We say that the shape divides the surface of the blackboard into two **regions**, one inside the shape, the other outside the shape.



This is true for all the shapes in the top left-hand set on page 18.

If now we draw one of the shapes in the right-hand set we have more than two regions.



For the shape shown we see that there are three regions, two within the shape and one outside it.

For the other shape in the right-hand set there are six regions. Five are enclosed by the shape itself. The other is outside it.

Using this idea we could describe the left-hand set as closed shapes each of which encloses only one region. The right-hand set are closed shapes each of which encloses more than one region.

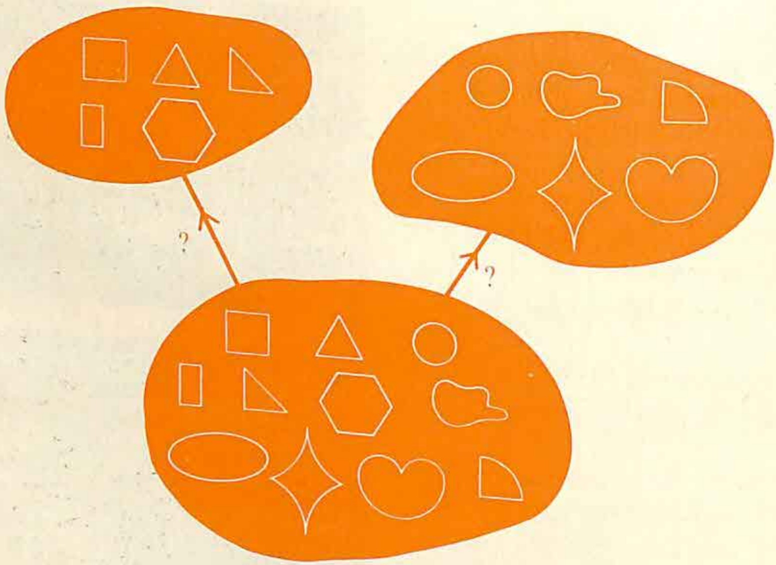
B. Closed shapes with all straight edges

On page 18 our sorting formed two sets. Since shapes enclosing only one region are the simplest, we shall consider this set first. We can find out more about these shapes through further sortings.

Exercise

6. Suggest ways of sorting the set of shapes, shown on page 18, which enclose only one region.

A simple way of doing the sorting for Exercise 6 is shown below.



Exercise

7. For each of the arrowed lines in the above sorting suggest a possible phrase to write by it.

A closed shape with all straight edges is called a **polygon**.

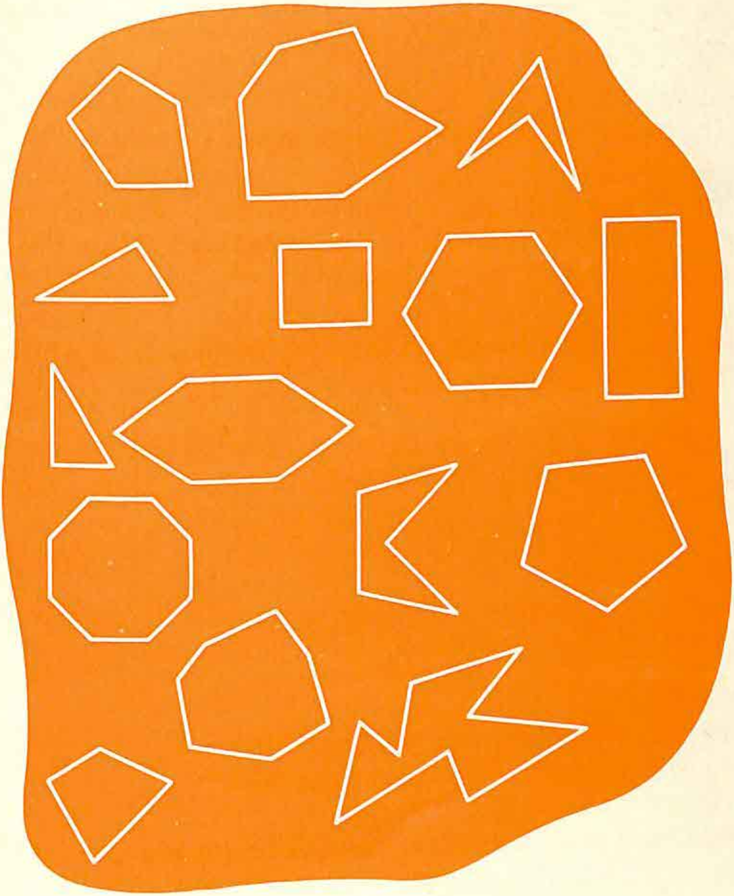
Exercise

8. Draw a set of polygons different from those shown above.

Once again we first choose to examine further the simpler of the two sets formed by our sorting on page 20; that is, the set of polygons.

In order to get the most out of this further work we will extend our set so as to include as many different polygons as we can within the space available.

Here is such a set:



Exercise

9. Suggest ways of sorting the set of polygons shown above. Use a sorting tree to show each of your sortings.

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SORTING 2-D SHAPES

One of the ways in which the set of polygons on page 21 can be sorted is to put them into sub-sets according to the number of their edges. To the polygons in each sub-set a special name is given, as follows:

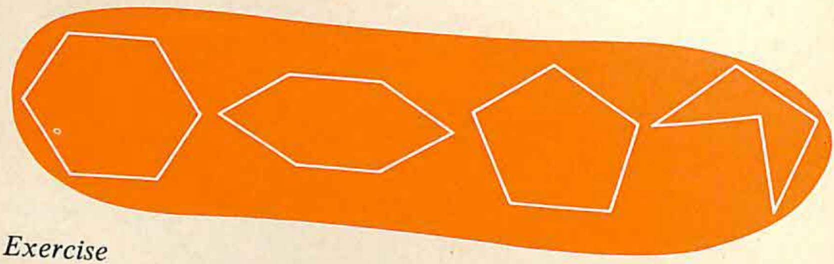
3 edges	**triangle**
4 edges	**quadrilateral**
5 edges	**pentagon**
6 edges	**hexagon**
7 edges	**heptagon**
8 edges	**octagon**
etc.	

Exercises

10. Try to find out why these various names are used.

11. The sorting could also be done on the basis of whether or not all the edges of a polygon are the same length. Show this, using a sorting tree.

Here are some of the shapes from the set on page 21 which have all their edges the same length.



Exercise

12. In what way do the two hexagons differ?
    In what way do the two pentagons differ?

A polygon with all its edges the same length and all its angles the same size is called a **regular** polygon.

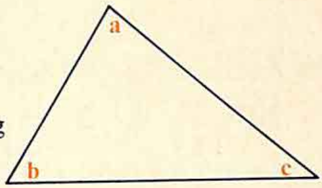
It may be worth noting at this stage that a polyhedron whose faces are all identical regular polygons and whose corners are all identical is called a **regular** polyhedron.
The cube is the regular polyhedron which we must frequently see.

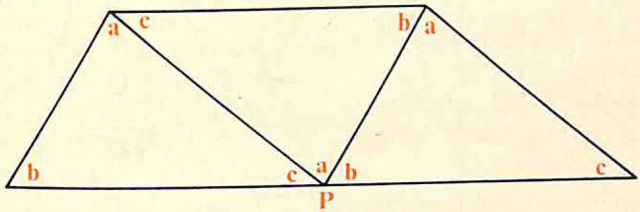
To be able to use regular polygons it is helpful if we know something about their angles.

The following exercises should lead to this knowledge.

Exercises

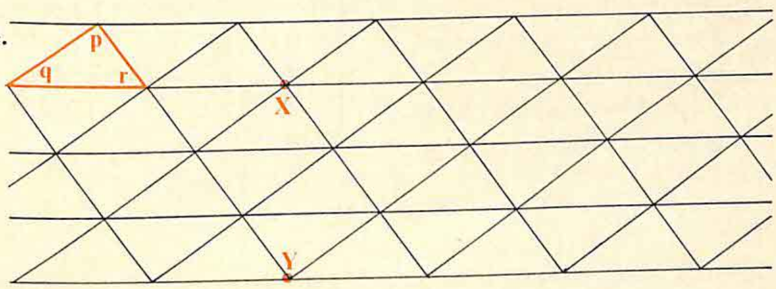
13. From card or thick paper cut out three identical triangles. For each, label the corresponding angles, a, b and c.





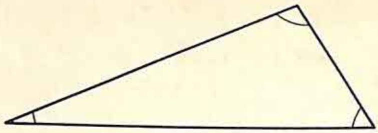
Arrange the three triangles as shown.
What can you say about the three angles a, b and c, at P?
Start with a different triangle and repeat the activity.
What can you say about the sum of the three angles of any triangle?

14.

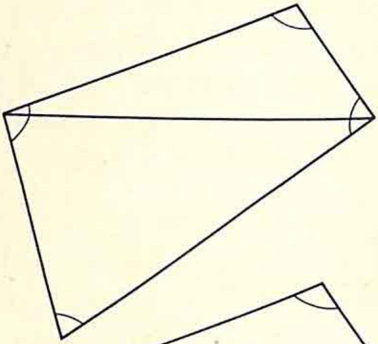


Use the coloured triangle to help you to label all the angles at the point X and at the point Y.
It is true to say that the sum of p, q and r is 180°?

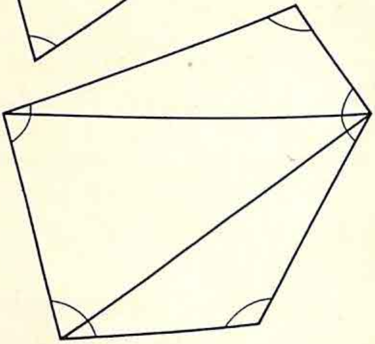
15. (a) Look at the triangle.
What is the sum of the three angles?



(b) Another triangle has been added to the first triangle to form a quadrilateral.
What is the sum of the angles of the two triangles?
What is the sum of the four angles of the quadrilateral?



(c) Another triangle has been added to the quadrilateral to form a pentagon.
How many triangles are there now?
What is the sum of the five angles of the pentagon?



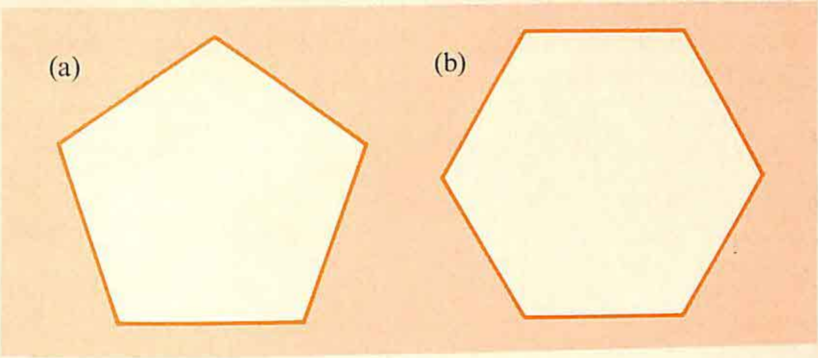
(d) Go on adding more triangles, one at a time.
Find the angle sum of each polygon formed.
Record your results in a table, as shown.

Number of edges	Number of triangles	Angle sum
3	1	180°
4	2	360°
5	3	540°
6		
7		
8		

16. Look at your table of results for Exercise 15.
Can you see a relationship between the number of edges and:
(a) the number of triangles? (b) the angle sum?

17. What is the angle sum of a polygon with:
(a) 10 edges? (b) 12 edges? (c) 20 edges? (d) 50 edges?

18. Write down the angle sum of the regular polygon:

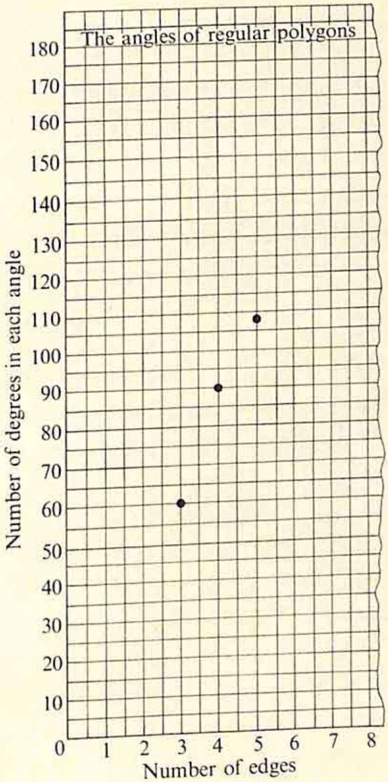


Find, in degrees, each angle of the pentagon and of the hexagon.

19. Part of a graph is shown on the right.
Copy it, extending the number of edges to 20.

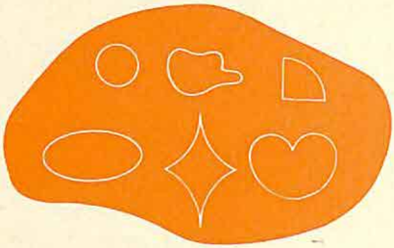
Show the size of an angle for each polygon.

Do you think the angle size will ever reach 180°?



C. Closed shapes with not all straight edges

In our sorting of closed shapes on page 20 we formed a set of polygons and a set of shapes with not all straight edges. For ease of reference the second set is again shown here.



Exercise

20. Suggest ways of sorting the set of shapes shown above. Using a sorting tree to show each way.

Once again there are several ways in which this sorting can be done. One way could be to sort them into

(a) the set of shapes with only curved edges and

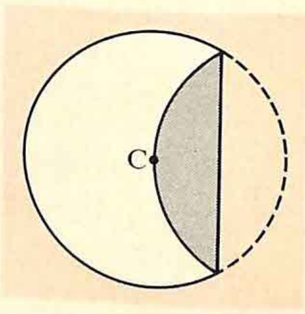
(b) the set of shapes with some curved and some straight edges.

We could then go on to find out more about the shapes in each of these two sets, but that is outside the scope of this book. It is perhaps of more value here to investigate ways in which some of these shapes can be formed.

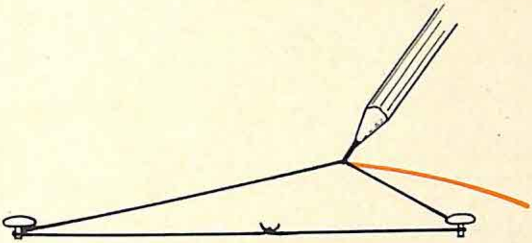
Exercises

21. Mark a point P on a sheet of paper. Now mark many points, each of which is 10cm from P. What shape is formed by the set of points?

22. Draw a circle, centre C, on paper (or tracing paper). Cut it out. Fold the paper so that the edge just reaches the centre as shown. Repeat many times for different fold lines, making sure each time that the edge just reaches C. Look at the fold lines. What do you notice?



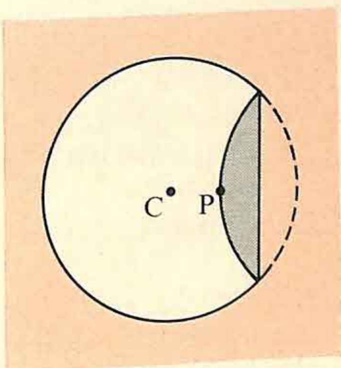
23. Fasten a piece of paper onto a drawing board or other flat surface. Stick two drawing pins, A and B, about 10cm apart, near the middle of the paper.
Make a loop of thin string about 25cm long and place it round the drawing pins as shown.



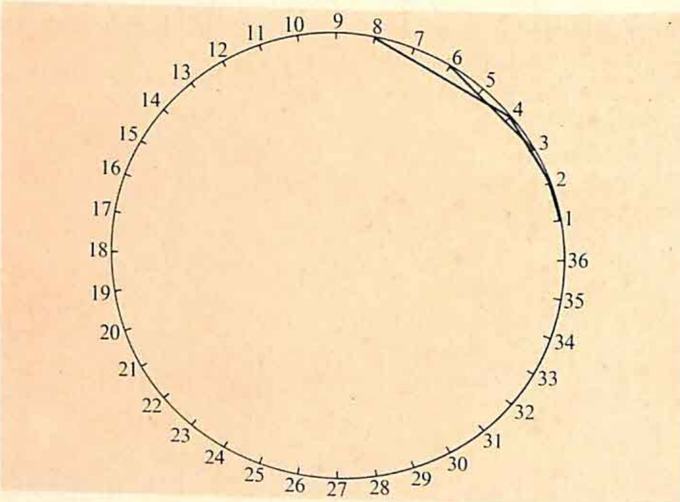
Put a pencil point in the loop and pull the string taut.
Keeping the string taut, move the pencil point on the paper.
Go on until you are back at your starting point.
The shape you obtain is called an **ellipse**.

Investigate what happens when the two pins are moved:
(a) further apart (b) nearer together.
What happens when A and B are at the same point, and only one of them is used?

24. Repeat Exercise 22, but this time mark a point, P, not at the centre of the circle.
Fold the paper so that the edge just reaches P, as shown.
Repeat for many other fold lines.
Look at the fold lines.
What do you notice?
What shape can you see?
Repeat the activity for other positions of P.



25. Draw a large circle (radius about 6 cm).
Mark points every 10° round the circle. Number the points 1 to 36.



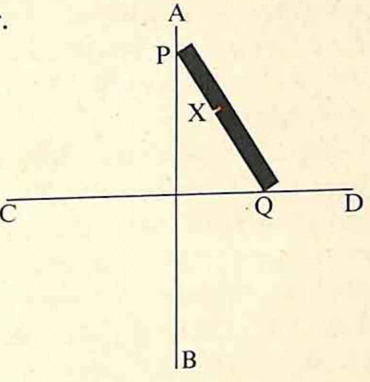
Draw a straight line from point 1 to point 2.
Then another straight line from point 2 to point 4.
Continue in this way to join 3 to 6, 4 to 8, 5 to 10, etc..
That is, join each point to its double.
The double of 19 is 38. This is not shown but it can be thought of as (36 + 2). So we join 19 to 2, 20 to 4, 21 to 6, etc..
Continue in this way until you get to 36.
The shape which is formed is called a **cardioid**.

Repeat the activity, but this time join 1 to 3, 2 to 6, 3 to 9, etc. That is join each point to its treble.
For this activity it is advisable to divide the circle into 48 equal parts.
The shape which is formed is called a **nephroid**.

Experiment with other rules for joining the points. For example, multiply the number of each point by four. For these it may be advisable to divide the circle into 72 equal parts.

26. Draw two lines AB and CD at right angles to each other. These lines should each be at least 30 cm long.

Use a strip of wood or card (or a short ruler) about 15 cm long. Mark a point X on this strip, 5 cm from one end.



Put the strip, PQ, so that P is on AB and Q on CD. On the paper mark the position of X.

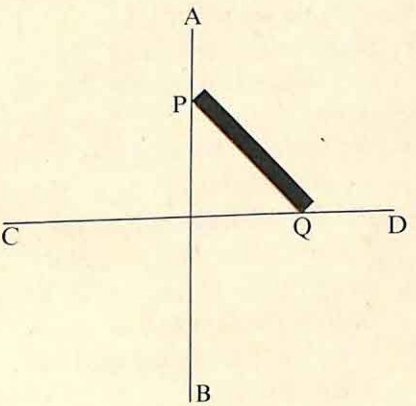
Move PQ to another position and again mark the position of X. Repeat for many other positions of PQ.

In turn repeat the activity in each of the other three spaces formed by AB and CD.

Join the set of marked points. What shape is formed? Repeat for other positions of X on PQ.

27. Use two straight lines and a strip of wood, as in Exercise 26.

This time however do not mark a point on the strip of wood. Instead, draw a line on the paper along the edge of the strip.



Repeat for many other positions of PQ.

Go on to draw sets of lines in the other three spaces.

The shape which is formed is called an **astroid**.

D. Closed shapes which enclose more than one region

Turn back to page 18. Look at the two sets of closed shapes. We have examined the set of shapes enclosing only one region. Now we turn our attention to the set of shapes which enclose more than one region.

These shapes are more complicated than most of the shapes we have been investigating so far. They are, however, equally important, though in a different way. They are the kind of shapes which we get when we are considering road systems, when we make maps, and when we represent electrical circuits.

For these, and other, purposes two new names are introduced.

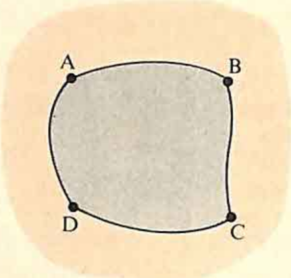
Look at the shape shown here.

Four points, A, B, C and D are marked.

From each of these points lines are drawn to other points.

Each of the four marked points is called a **node**.

Each of the four lines is called an **arc**.



The shape also forms two regions.

The four nodes can be thought of, for example, as four towns; the four arcs as roads; the two regions as states or countries.

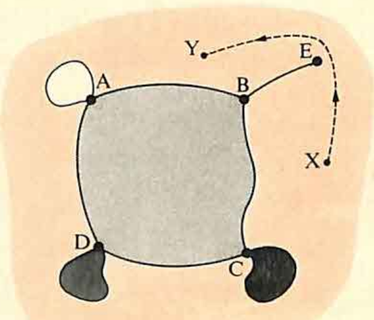
In this example we see that:

(a) Some of the arcs start and finish at the same node.

(b) the node E has only one arc going from it.

(c) the arc BE does not make a new region. It is possible to get from, for example, X to Y without crossing an arc.

(d) there are 5 nodes, 8 arcs and 5 regions.

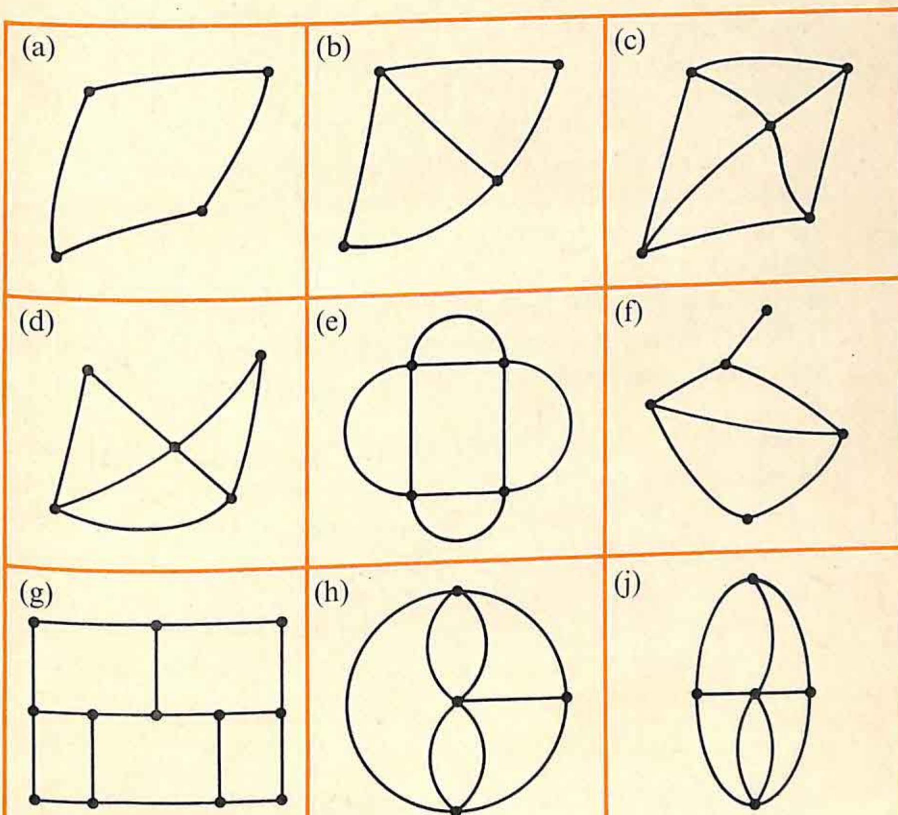


Shapes of this kind are often called **networks**.

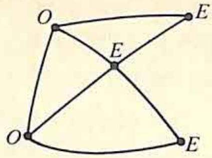
Exercises

28. Think of the network shown at the bottom of page 30 as a road system joining five towns.
    Is it possible to start at one of the towns and walk along all the roads without going along any road more than once?
    Is it possible to start and end such a journey at the same town?

29. Decide which of the shapes shown below can be drawn if you keep to the two rules:
    (i) *your pencil must not be taken off the paper*
    (ii) *no line must be drawn twice.*
    Look at those which you can draw. For which of these is it possible to start and end at the same node?



30. Look at the network shown here.
The number of arcs going from each node has been counted.
If the number is even, an *E* has been written at the side of the node.
If the number is odd, an *O* has been written.

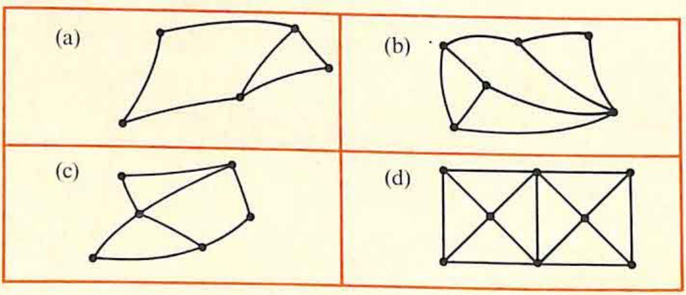


Copy the networks shown in Exercise 29 and write an *O* or an *E* against each node.
Show your results in a table, as below.

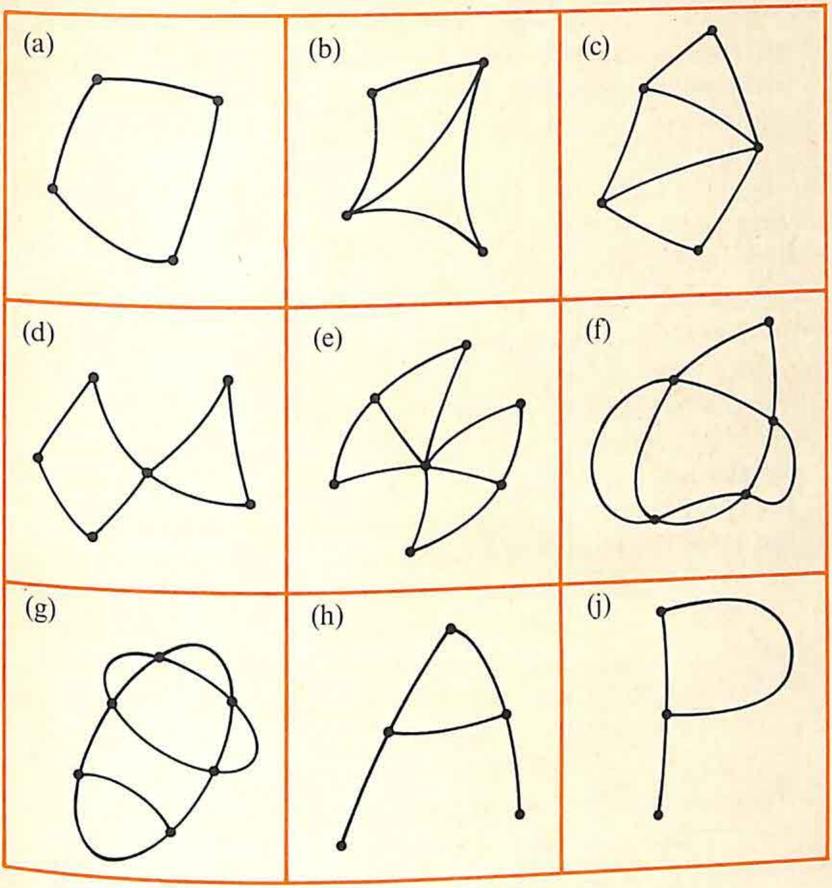
Network	a	b	c	d	e	f	g	h	j
Number of even nodes	4	2							
Number of odd nodes	0	2							
Can it be drawn? (Keeping the rules)	Yes	Yes							

31. The famous mathematician Euler was very interested in networks. He decided that, keeping to the rules stated on the previous page, a network can only be drawn if
either (a) it has all even nodes,
or (b) it has two, and only two, odd nodes.
Is Euler's statement true for the networks you have examined?
Try to draw a network for which Euler's statement is not true.

32. Using Euler's statement decide which of these networks can be drawn (keeping to the rules). Then draw them.



33. For each of these networks count the nodes, arcs and regions. Show your results in a table, as shown below.



Shape	a	b	c	d	e	f	g	h	j
Number of nodes (N)									
Number of regions (R)									
Number of arcs (A)									

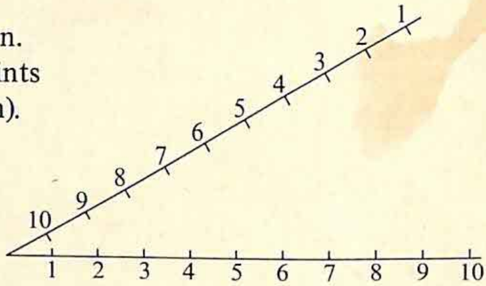
34. Look at your table of results for Exercise 33. Can you see a relationship between the numbers of the nodes, regions and arcs? If so, compare it with the relationship you found on page 12. What do you notice?

E. Open shapes

In our first sorting of 2-D shapes on page 17 we separated the set of shapes into closed shapes and open shapes. In our work since then we have considered only the closed shapes. We should now go on to examine the open shapes. But, again, at this stage, this work is not appropriate. It is better for the time being to investigate ways in which some of these shapes are formed.

Exercises

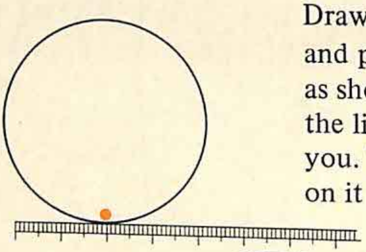
35. Draw two lines, as shown. Mark equally spaced points on each line (at least ten). Number the points.



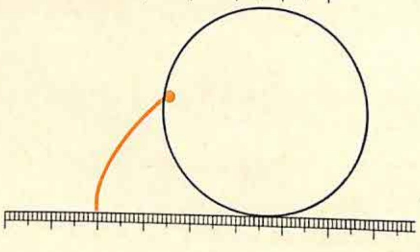
Join the two points, labelled 1, with a straight line.

Then join the two points labelled 2. Continue in this way. The shape formed is a **parabola.**

36.



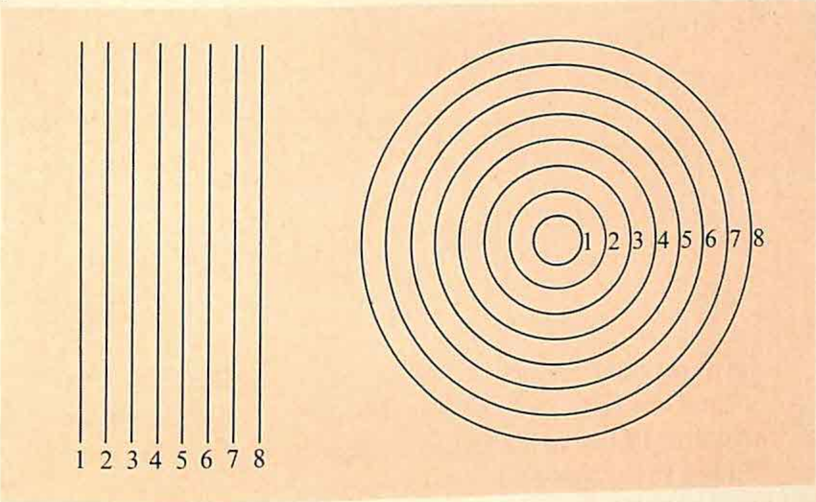
Draw a straight line on a sheet of paper and place a large coin flat on this line as shown. Place the edge of a ruler along the line and ask a friend to hold it for you. Turn the coin until a marked point on it touches the line as shown.



Now slowly roll the coin along the ruler and from time to time mark the position of the point. Join the set of marks by a smooth curve.

The shape obtained is a **cycloid.**

37.



Draw a set of twelve (or more) equally spaced circles (eight of which are shown above). Number the circles.
On a piece of tracing paper draw the same number of parallel lines with the same spacing as the circles. Number the lines.

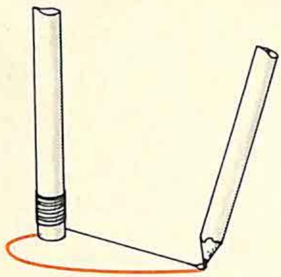
Place the tracing paper over the circles. Look at the points where a circle cuts a line with the same number.
Do all the circles cut a line with the same number?
Mark the points of intersection of those that do cut.
Join the marked points. The shape you obtain is a parabola.

Change the position of the tracing paper and repeat the activity.

38. On another piece of tracing paper draw a set of parallel lines as in Exercise 37 but this time make the equal spaces greater than those between the circles.
Repeat Exercise 37. What shape do you obtain?

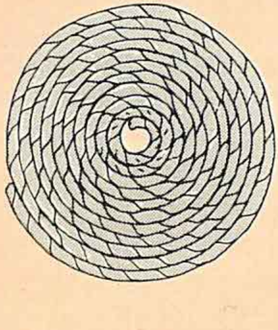
39. Again repeat Exercise 37 but make the spaces between the parallel lines less than those between the circles.
The shape which is formed is called a **hyperbola**.

40.



Tie one end of a piece of thin string round one end of a pencil. Make a small loop at the other end of the string. Now wrap the string *tightly* round the end of the pencil. Stand the pencil on a sheet of paper. Put a pencil point through the loop in the string. Keeping the string taut slowly unwind it from the pencil. As you unwind, mark the paper with the pencil which is in the loop. The shape formed is a **spiral.**

41. Here are some examples of spirals we see in everyday life.

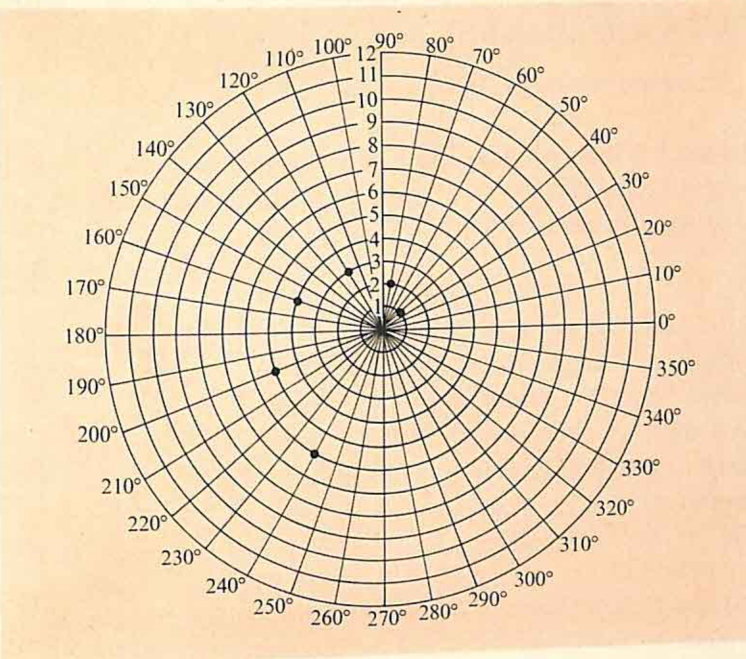


a coil of rope

the groove on a gramophone record

Find and draw some other examples of spirals.

42.



Draw a set of equally spaced circles (larger than above).
Number them, then draw lines every 10°, as shown.
Mark the point where circle '1' cuts the 40° line.
Mark the point where circle '2' cuts the 80° line.
Go on in this way, marking the points (3, 120°), (4, 160°), (5, 200°) etc. Join the marked points with a smooth curve.
What shape do you get?

Mark the set of points, (1, 30°), (2, 60°), (3, 90°), ...
Mark other sets of points, choosing your own rules for them.

Important ideas in this chapter

1. The idea of open and closed shapes.
2. The idea of a polygon.
3. The sum of the angles of a polygon.
4. The idea of a network.
5. The relationship between the numbers of nodes, arcs and regions.
6. Some important open and closed shapes.

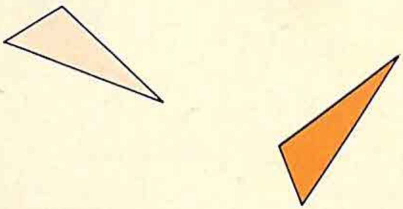
5. MOVING SHAPES

A. Changing the position of a shape

In the work which we have done with shapes so far we have kept them stationary. We are now going to look at some plane shapes and see what happens when they are moved.

Exercise

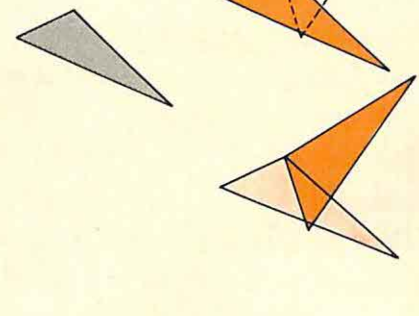
1. From card cut out a triangle like the red one shown. Place the triangle on a sheet of plain paper. Mark its outline. Pick it up and move it somewhere else on the paper. Mark the new outline.



   Put the triangle back in its first outline. Move it to the second outline, keeping it flat on the paper all the time. Find out all you can about possible ways of moving the triangle from the first outline to the second.

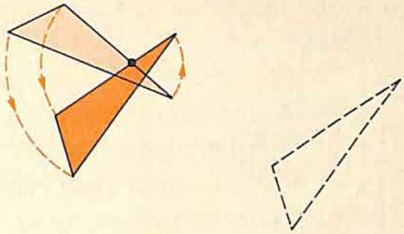
The exercise which we have just completed will have emphasised the many ways in which such a movement can be made. Here are two of the ways:

(i) Move the triangle in a straight line until one of its vertices is coincident with the corresponding one on the second outline.

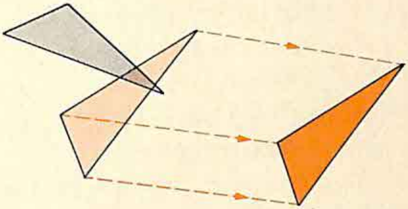


Now, keeping that vertex in the same position, turn the triangle until it fits into the outline.

(ii) First turn the triangle about a fixed point until each of its edges is parallel to the corresponding edge on the second outline.



Now move the triangle in a straight line until it fits into the second outline.

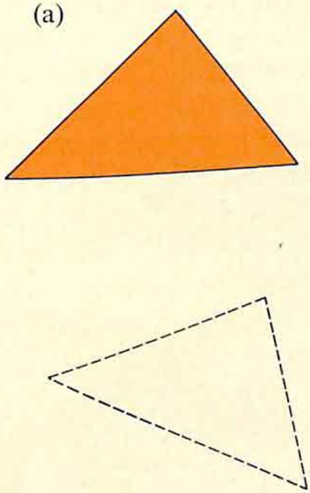


In each of these illustrations, we have completed the movement by combining two simple movements. In (i) we did a straight line movement first. Next we did a turn. In (ii) we did a turn first and followed it with a straight line movement.

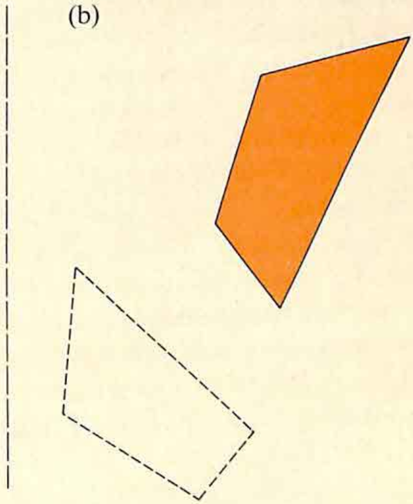
Exercises

2. Show several ways of moving the shape to the second (dotted) outline. Use at least one combination like (i) opposite and one like (ii) above.

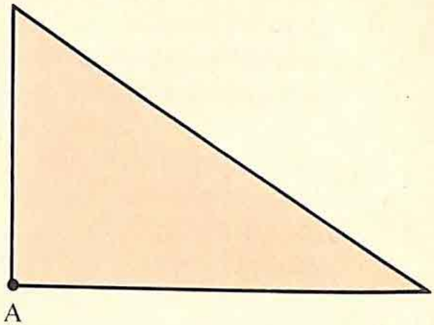
(a)



(b)



3. Copy this shape and show the result of each of the movements listed.

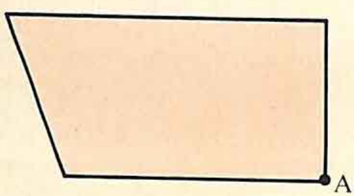


   (i) Move 10 cm to the right along a line parallel to the bottom of your paper.
   (ii) Turn through one right angle in an anti-clockwise direction about the vertex A.
   (iii) First (i) then (ii)
   (iv) First (ii) then (i)

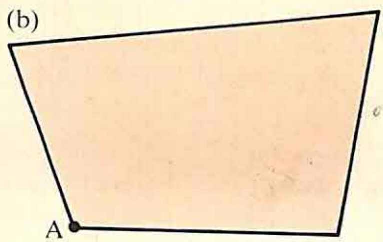
   What do you notice about (iii) and (iv)?

4. Repeat the activity of the previous exercise using

(a)



(b)



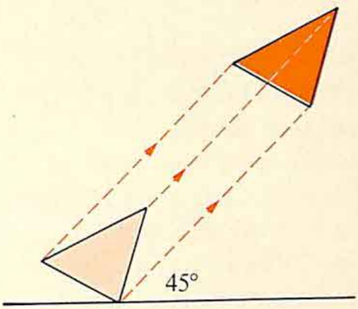
   What can you say about the results of (iii) and (iv) for each of the shapes?
   What general rule is indicated by your results?

The straight line movement and the turning movement are both very important. We have seen how we can combine them to give other movements.
We have also seen that we can take many movements and replace them by combinations of these two more simple ones.
As we shall use these two movements frequently, we shall use the special names which they are often given.
The straight line movement is called a **translation.**
The turning movement is called a **rotation.**

B. Describing the movement: translation

To describe a translation we have to give two pieces of information.

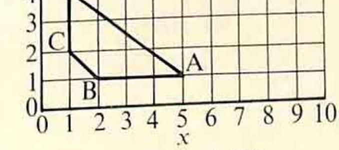
First we have to say in which direction the translation is to take place.
Second we have to say how far the shape is to move.
For example, we might describe a translation as *move along a line at 45° to the bottom edge of the paper for 4 cm.*



Exercises

5. From card cut a square of edge 4 cm. Place it on a sheet of plain paper with one edge parallel to the bottom of the paper. Mark the outline of the square.
   Move the square using the translation *5 cm to the right parallel to the bottom of the page.* Mark the new outline of the square. Now move the square using the translation *8 cm up parallel to the edge of the page.* Mark the outline of the shape.
   Describe the single translation which produces the same result as the two translations which were used.

6. On a piece of squared paper draw a pair of coordinate axes as shown.
   Cut out a copy of the quadrilateral from card.
   Place the cut-out shape on the squared paper as shown.



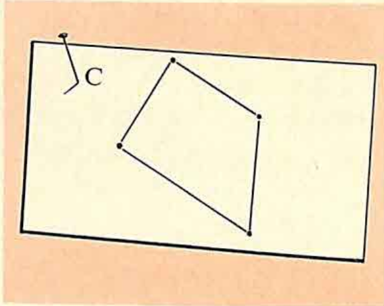
   (a) Now move the quadrilateral by a translation until the vertex A is at (9,1).
   What are the new coordinates of vertices B, C, and D?
   (b) Follow this first translation with a second which takes the vertex A on from (9,1) to (9,6).
   Where are B, C and D now?
   (c) The two successive translations have produced a result which could have been made by a single translation. Describe it.

C. Describing the movement: rotation

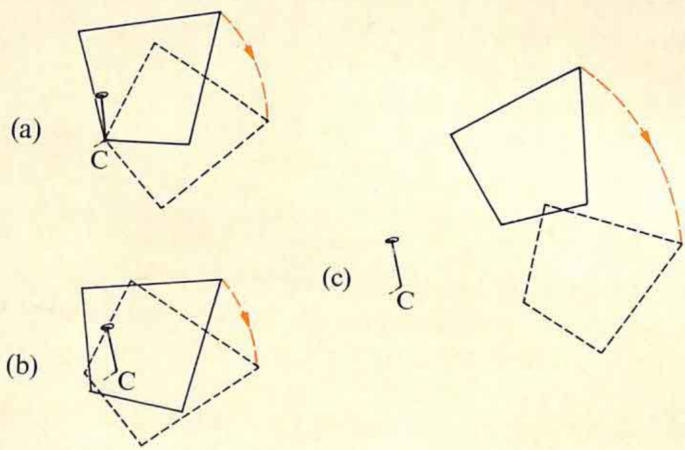
We now turn to the second basic movement, rotation.

Exercise

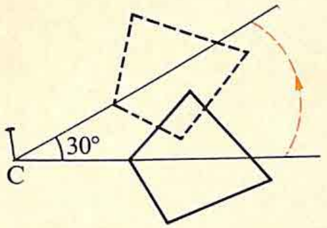
7. Draw a quadrilateral on a post-card and mark a point C as shown. Place the card on a piece of plain paper. Put a pin through C so that the card can be moved around this point. Use another pin to prick the positions of the vertices of the quadrilateral through to the paper. Rotate the card about C through approximately one right angle. Prick through the new positions of the vertices. Remove the card and on the paper mark both positions of the quadrilateral. Mark the position of C, the centre of rotation. Repeat the activity using other positions of C. Some of these should be within the quadrilateral and some on its edges.



You will have found that there are two important things we need to know about a rotation. One of them is the position of the centre of rotation. Here are some rotations of the same shape using different centres of rotation:

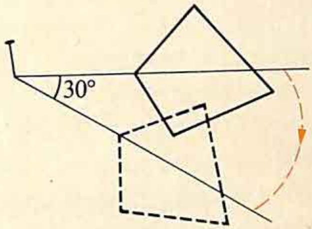


The second thing we need to know about a rotation is its size. This can be described by stating the angle the shape turns through and its direction: clockwise or anticlockwise.



A rotation of 30° in an anti-clockwise direction.

A rotation of 30° in a clockwise direction.



Exercises

8. In this exercise you will need polar graph paper like that drawn on Worksheet 1.

   Place the post-card from Exercise 7 on a sheet of the paper. Put a pin through the point marked C on the post card and stick the pin-point into the centre of the polar graph paper.

   Holding the card still, prick through the vertices of the quadrilateral with another pin.

   Make a rotation of the quadrilateral about C. Prick the new position of its vertices through to the graph paper.

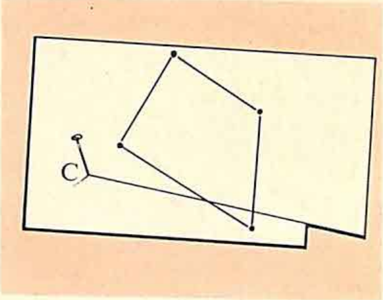
   Use the markings on the graph paper to describe the rotation. State its size and whether it is clockwise or anticlockwise.

   Repeat the activity using other centres of rotation on the same post card, always pinning them to the centre of the paper.

   Also try other post cards with different shapes on them.

   Repeat the activity using several different centres of rotation for each post card.

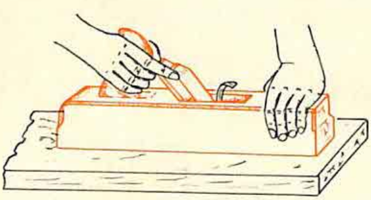
9. We shall again use the card of Exercise 7 in this exercise.
Draw a straight line from C to an edge of the card. Also cut out a small piece of the card, as shown.

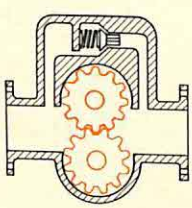


Carry out a rotation of the quadrilateral on polar graph paper marking
(a) the original position of the quadrilateral and of the line through C.
(b) the final position of the quadrilateral and the line through C.
What can you say about the angle of rotation and direction of rotation of the line and of the quadrilateral?
Repeat the activity for other rotations.
How can the line be used to make specified rotations of the quadrilateral?

10. Use the method of the previous exercise to find the single rotation which is equivalent to the rotation
(a) 40° clockwise followed by 50° clockwise;
(b) 70° clockwise followed by 30° anticlockwise;
(c) 60° anticlockwise followed by 110° clockwise.

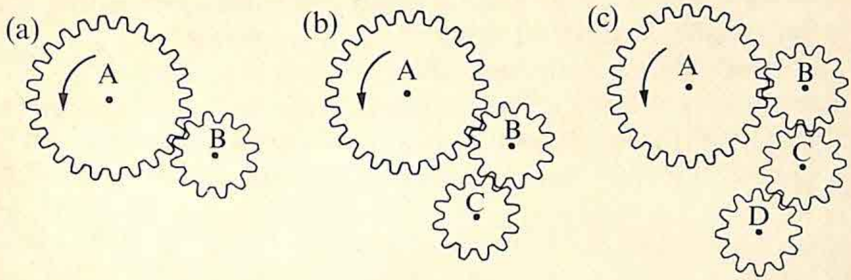
11. We often make use of simple translations and rotations in everyday life. For example, the plane moves by a translation and the cog-wheel by rotation.





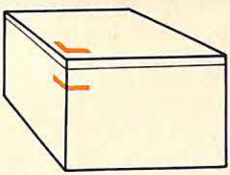
List other examples of everyday uses of translations and rotations.

12. Here are some sets of cog-wheels. The large cogs are all identical. So too are the small ones. The number of teeth on a large cog is double the number on a small one.

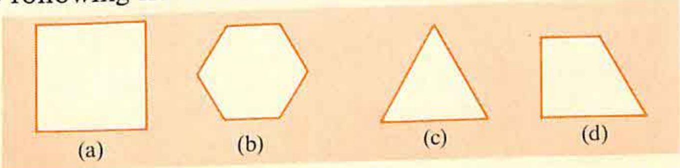


In each case show the direction of rotation of the other cogs as A turns anticlockwise.
How many times will each cog rotate when cog A in each set makes one complete turn?

13. Obtain a biscuit tin or other container which is a cuboid. Mark a corner of the lid and the corresponding corner of the container as shown.
Lift the lid just above the container and rotate it slowly until it will fit on again.
What do you notice about the mark on the lid and that on the container?
Lift the lid and continue rotating slowly until it fits again.
Where are the two marks now?
How many times in one complete rotation does the lid fit?



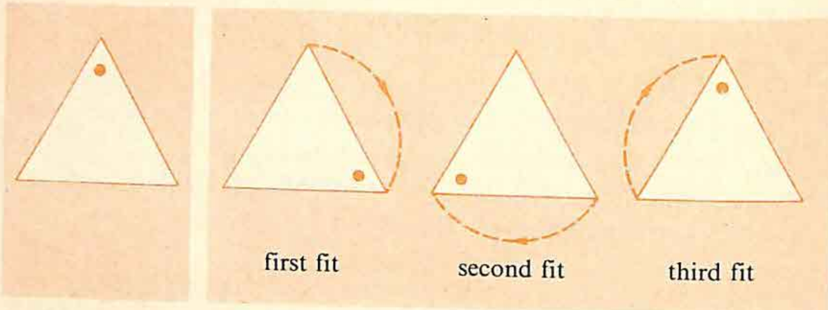
14. How many times in a complete turn would a lid shaped like the following fit its container?



15. Draw or make a lid which, in one complete turn, would fit its container (a) 5 times (b) 8 times.

D. Rotational symmetry

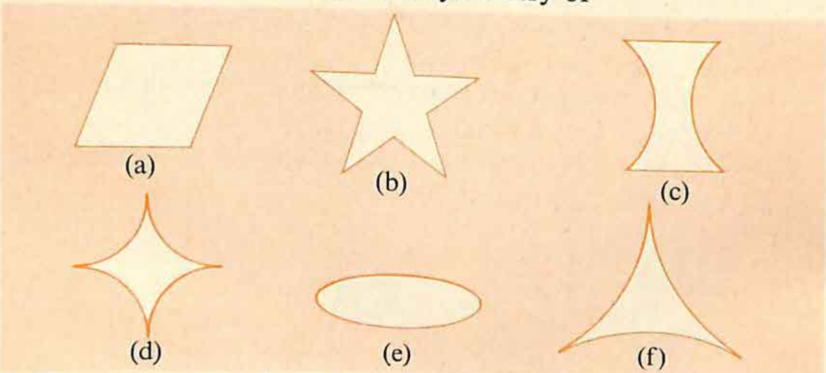
When a plane shape fits into its outline more than once in a complete turn we say that it has **rotational symmetry**.
We have seen that a rectangle, a square, an equilateral triangle, and a regular hexagon all have rotational symmetry.
However, there is something different about each of these rotational symmetries. This difference is the number of times each shape fits into its outline in a complete turn. The equilateral triangle, for example, fits three times.



The number of times a shape fits its outline in a complete turn is called its **order of rotational symmetry**. For the triangle the order is 3.

Exercises

16. What is the order of rotational symmetry of



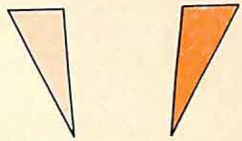
17. Draw or make a plane shape with rotational symmetry of order
(a) 3 (b) 4 (c) 5 (d) 7
Do not use any of the shapes which we have already examined.

E. Reflections

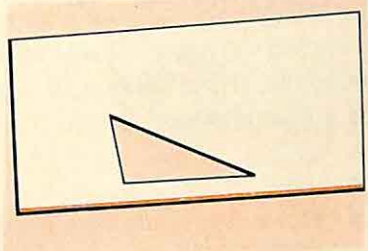
We have seen that we can move a plane shape to any new position using only translations and rotations. Is this always so? Consider the example of Exercise 18.

Exercises

18. Try to find a way of moving the triangle to the second outline using only translations and rotations.



19. On a post-card draw a triangle like the one shown.



Carefully cut the triangle from the card to leave an outline.

We shall be using the outline triangle in this exercise.

Place the card on a plain sheet of paper. Mark the outline of the triangle.
Now turn the card over keeping the red edge still and in contact with the paper.
Again mark the outline of the triangle.

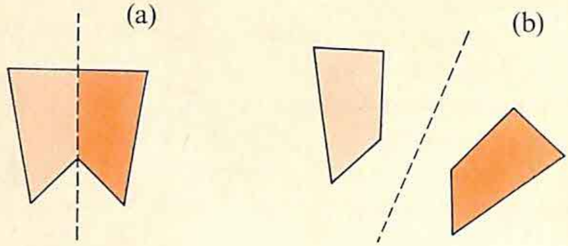
How does your result compare with the diagram in Exercise 18?

Can you move the triangle from its first position to its second one, using only translations and rotations?

20. Repeat the activity of Exercise 19 using as the fixed line
    (a) another edge of the card,
    (b) a new edge of the card made by cutting off a corner.

These exercises show us that translations and rotations alone are not sufficient to deal with all the possible movements in a plane. The turn-over which we have now introduced allows us to make many other movements.

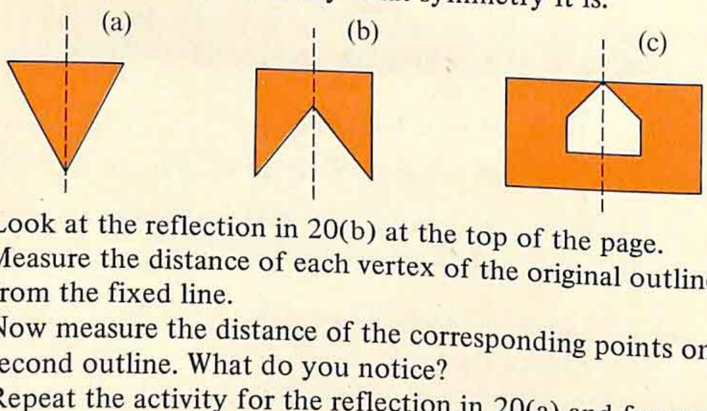
We have also seen that when we use a turn-over it is important to select the fixed line carefully. The line may be an edge of the shape as in (a) below. It may be any other line as in (b).



Instead of calling this movement a turn-over, the word reflection is commonly used. This is because the shape in its new position looks like the reflection of the original which would be formed by a mirror placed on the fixed line.

Exercises

21. These shapes are produced by putting together the first and second outlines for a reflection. Has the combined shape any kind of symmetry? If so say what symmetry it is.



22. Look at the reflection in 20(b) at the top of the page. Measure the distance of each vertex of the original outline from the fixed line.
    Now measure the distance of the corresponding points on the second outline. What do you notice?
    Repeat the activity for the reflection in 20(a) and for several other reflections. What do you notice?

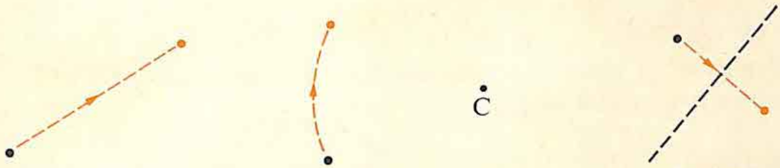
Important ideas in this chapter

1. The idea of translation, rotation, and reflection.
2. The idea of rotational symmetry.

6. SIMILARITY AND CONGRUENCE

A. Moving individual points

In Chapter 5 we moved shapes such as triangles or quadrilaterals using translations, rotations and reflections. We can, of course, carry out any of these movements with single points, as below.

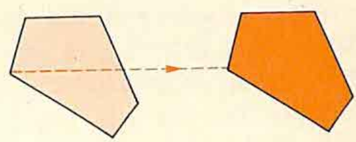


a translation a rotation a reflection

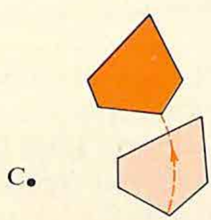
We often make use of movements of individual points when we have a shape to translate, rotate or reflect.

Exercises

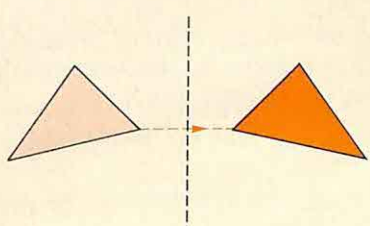
1. Here is a translation of a pentagon. Describe the movement of the individual points of the pentagon during the translation.



2. Here is a rotation of a pentagon. Describe the movement of the individual points of the pentagon during the rotation.



3. Here is a reflection of a triangle. Describe the movement of the individual points of the triangle during the reflection.

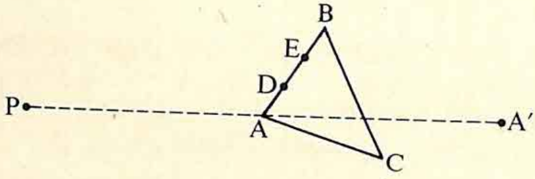


B. Similar shapes

The movements we considered in Chapter 5, translations, rotations, and reflections, always left the size of the shape unaltered.
Only its position was changed.
We shall now use another movement. Here we must deal with each individual point of the shape separately.

Exercise

4. On a sheet of plain paper draw a triangle, and mark a point P as shown on the diagram.



Draw a line from P to A and then extend it to a point A′, so that the distance from P to A′ is twice the distance from P to A. In the same way extend a line from P to B to a point B′ so that the distance PB′ is twice the distance PB. Carry out the same procedure for point C to locate point C′, for point D to locate point D′, and for point E to locate point E′. If this procedure were repeated for every possible point of the triangle ABC, what shape would be formed?

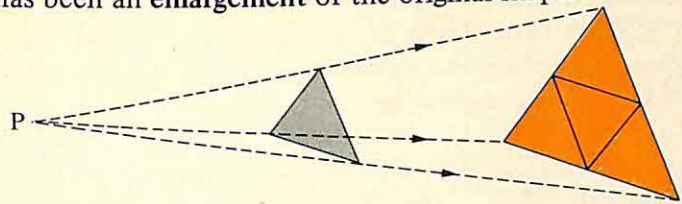
In Exercise 4 we found a new point for each point of the original triangle. It is helpful to think of the result as a movement of the original points. We think of a point being moved along the line from P through A until it reaches A′ where its distance from P is doubled. Each point of the original triangle is moved in the same way.

Exercises

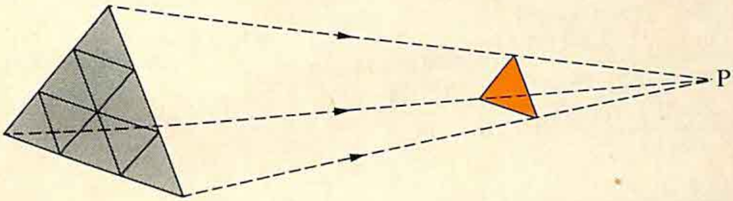
5. Repeat Exercise 4 moving each point until its distance from P is (a) trebled, (b) quadrupled, (c) halved.
What can you say about (i) the shape formed and (ii) its size?

6. Repeat the activities of the previous exercises starting with a quadrilateral rather than a triangle.

Unlike the previous movements, this type brings about a change in the size of the shape. We see that if the rule which we use increases the distance of the individual points from P we get an increase in size. This situation is shown below. We say that there has been an **enlargement** of the original shape.

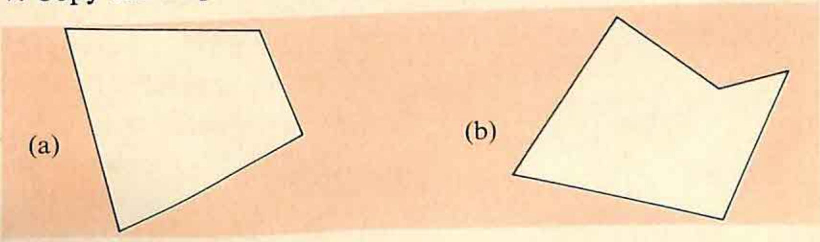


In the diagram the distances from P have been doubled. We see that the new shape has an area four times the original.
When the distance of the individual points from P is decreased we also have a decrease in area. We say that there has been a **reduction.**



In the diagram the distances from P have been reduced to one third. We see that the area is reduced to one ninth of the original.
In the diagrams above all the triangles are the same shape. Such triangles are said to be **similar.**
In general, objects which are the same shape are similar. It does not matter if one is an enlargement or a reduction of the other or if they are the same size.

Exercises
7. Copy the shape. Use the method above to make similar shapes.



8. Use a tessellation of triangles like that on page 23 to pick out triangles similar to the red triangle.

9. On page 18 of Book 3 in this series there is a tessellation of equilateral triangles. Copy this tessellation.
Pick out a regular hexagon from the tessellation. Now pick out several similar hexagons. Repeat this activity by picking out other sets of similar polygons.

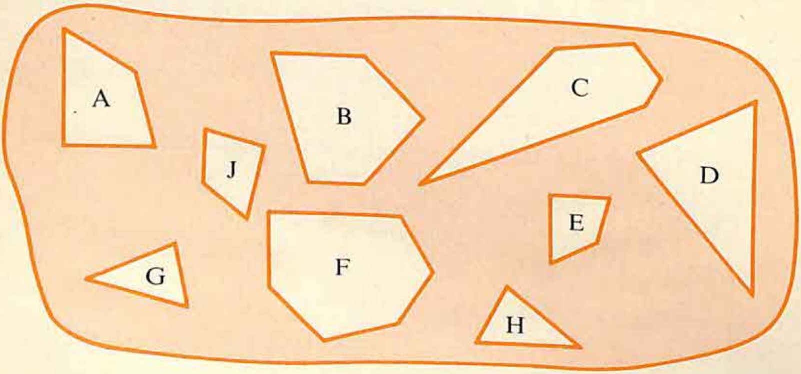
10. The diagrams near the top of page 51 show that
   (a) when the distance from P is doubled, edge lengths are doubled and areas are quadrupled.
   (b) when the distance from P is reduced to one third, edge lengths are reduced to one third and areas to one ninth.

   Copy and complete this table:

Distance from P is multiplied by	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	2	3	4	5
Edge length is multiplied by		$\frac{1}{3}$		2			
Area is multiplied by		$\frac{1}{9}$		4			

   Show the last two lines of this table on a graph.
   What is the shape of the graph?

11. From this set of objects pick out those which are similar.



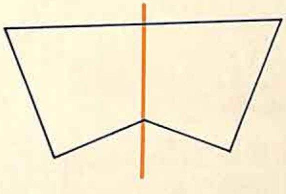
C. Congruent shapes

In the previous exercise the quadrilaterals E and J are similar. They are also the same size. Shapes which are the same size as well as similar are said to be **congruent**.

Other congruent shapes in Exercise 11 are the triangles G and H.

We have seen many examples of congruent shapes. In every translation the original and final outlines are congruent. This is also true for every rotation and for every reflection.

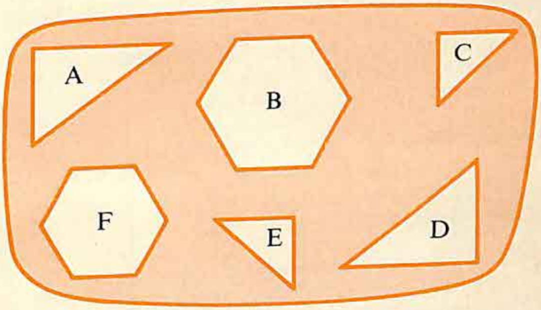
As every reflection leaves us with a pair of congruent shapes, we realise that all shapes with a line of symmetry must be composed of a pair of congruent shapes.



The shape in the diagram consists of two congruent quadrilaterals.

Exercises

12. Which of the shapes in this set are
    (a) similar?
    (b) congruent?



13. One use of enlargement is in the projection of a film onto a screen. List any other uses of enlargement you can think of.
14. One use of reduction is in producing a photograph. List any other uses of reduction you know of.

Important ideas in this chapter

1. The idea of moving individual points
2. The idea of enlargement and reduction.
3. The idea of similarity.
4. The idea of congruence.

ANSWERS

2. EQUIVALENT SHAPES

2. (a) saucepan, orange, fork, concrete block
   (b) cup, metal nut, needle, pipe
   (c) chair, car steering wheel, vase, bag
   (d) all the statements are correct.

3. Yes

4. Yes

5. Yes

7. (a) {knife, bottle, spring}
   (b) {key, ring}
   (c) {shoe, wheel}

3. SORTING 3-D SHAPES

3. From left to right, in order, the labels on the arrowed lines are: 'four faces'; 'five faces'; 'six faces'; 'seven faces'.

4. From left to right, in order,the labels on the arrowed lines are 'six edges'; 'eight edges'; 'nine edges'; 'twelve edges'; 'fifteen edges'.

5. From left to right, in order, the labels on the arrowed lines are: 'four vertices'; 'five vertices'; 'six vertices'; 'seven vertices';'eight vertices'; 'ten vertices'.

7. Yes

8. Yes

4. SORTING 2-D SHAPES

2. The labels on the arrowed lines could be, for example, 'shapes with no loose ends' and 'shapes with loose ends'.

5. The labels on the arrowed lines could be, for example, 'shapes without loops' and 'shapes with loops'.

7. The labels on the arrowed lines could be, for example, 'shapes with all straight edges' and 'shapes with not all their edges straight'.

12. The left-hand hexagon has all its angles equal. The other has not.
    The left-hand pentagon has all its angles equal. The second has not.

13. It seems that the angles a, b and c at P together make two right angles (180°).
    It seems that the sum of the three angles of any triangle is two right angles.

14. Yes

15. (a) 180°
    (b) 360°
        360°
    (c) There are now 3 triangles.
        540°
    (d)

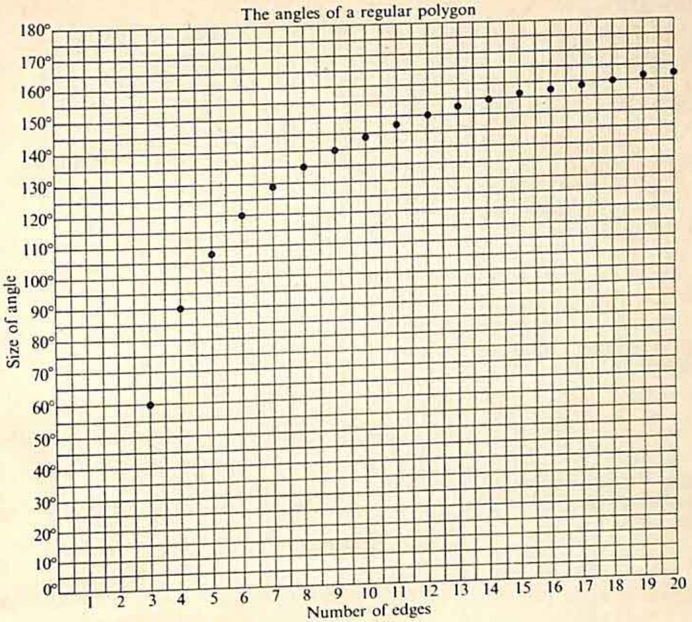
Number of edges	Number of triangles	Angle sum
3	1	180°
4	2	360°
5	3	540°
6	4	720°
7	5	900°
8	6	1080°
9	7	1260°
10	8	1440°

16. (a) The number of triangles is two less than the number of edges.
    (b) The angle sum is
        (the number of edges $-2) \times 180°$.

17. (a) 1440°
    (b) 1800°
    (c) 3240°
    (d) 8640°

18. (a) 540°; 108°
    (b) 720°; 120°

19.



The angle size gets closer and closer to 180° as the number of edges increases. It never gets to 180° however.

21. A circle.

22. The fold lines form a circle.

23. When only one pin is used a circle is formed.

24. The shape formed is an ellipse.

26. The shape formed is an ellipse.

28. Yes
No

29. (a), (b), (d), (e), (h), can be drawn keeping to the rules.
(a), (e) can be started and finished at the same node.

30.

Network	a	b	c	d	e	f	g	h	j
Number of even nodes	4	2	1	3	4	1	4	2	1
Number of odd nodes	0	2	4	2	0	4	8	2	4
Can it be drawn (keeping the rules)?	Yes	Yes	No	Yes	Yes	No	No	Yes	No

31. Yes, Euler's statement is true.

32. (a), (c) can be drawn.

33.

Shape	a	b	c	d	e	f	g	h	j
Number of nodes	4	4	5	6	7	5	6	5	3
Number of regions	2	3	4	3	5	6	7	2	2
Numbers of arcs	4	5	7	7	10	9	11	5	3

34. $N + R = A + 2$

38. The shape formed is an ellipse.

42. The shape formed is a spiral.

5. MOVING SHAPES

3. The overall results for (iii) and (iv) are identical.

4. For each shape the overall results for (iii) and (iv) are identical.
When a shape undergoes a translation and a rotation it does not matter which is done first. The final result is the same.

5. The single translation is '9·5 cm at an angle of $58^\circ\ 30'$ to the bottom of the page, the angle being measured in an anticlockwise direction'.

6. (a) B is at (6,1); C is at (5,2); D is at (5,4)
(b) B is at (6,6); C is at (5,7); D is at (5,9)
(c) 6·4cm at an angle of $51^\circ 20'$ to the x-axis, the angle being measured in an anticlockwise direction.

9. The angle of rotation of the line and the quadrilateral are the same. So too are their directions of rotation.
When a particular angle of rotation for the quadrilateral is required it can be obtained by turning the line through the same angle.

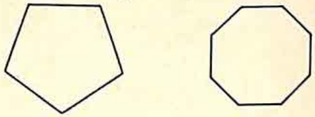
10. (a) 90° clockwise.
(b) 40° clockwise.
(c) 50° clockwise.

12. (a) B clockwise. 2 rotations
(b) B clockwise. 2 rotations
C anticlockwise. 2 rotations
(c) B clockwise. 2 rotations
C anticlockwise. 2 rotations
D clockwise. 2 rotations

13. Twice

14. (a) Four times
(b) Six times
(c) Three times
(d) Once

15. Here is an example of each



16. (a) 2 (b) 5 (c) 2
(d) 4 (e) 2 (f) 3

19. The result is like that in Exercise 18. The movement can not be carried out using only translations and rotations.

21. Line symmetry in all three cases. The line of reflection is the line of symmetry.

22. The distance from the fixed line of a particular point before reflection is the same as the corresponding distance after reflection.
This is true for all points on a shape undergoing reflection

6. SIMILARITY AND CONGRUENCE

1. Each individual point moves the same distance in the same direction.

2. Each individual point rotates through the same angle in the same direction and about the same centre of rotation.

3. Each individual point moves to the same distance on the opposite side of the line of reflection.

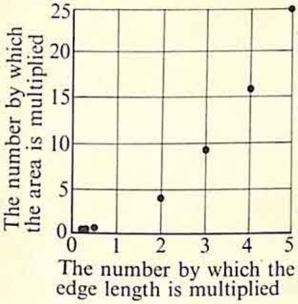
4. A triangle would be formed. It would be the same shape as the original triangle but four times as big.

5. (i) Each time a triangle having the same shape as the original is formed.
(ii) Their sizes are
(a) nine times that of the original triangle,
(b) sixteen times that of the original triangle,
(c) a quarter that of the original triangle.

6. Each time a quadrilateral, having the same shape as the original quadrilateral, is formed. The size is increased or decreased in the same way as for the triangle of the previous exercises.

10.

Distance from P is multiplied by	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	2	3	4	5
Edge length is multiplied by	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	2	3	4	5
Area is multiplied by	$\frac{1}{16}$	$\frac{1}{9}$	$\frac{1}{4}$	4	9	16	25



The graph is part of a parabola

11. A, E and J are similar.
D, G and H are similar.

12. (a) Similar shapes are: A, C, D and E; B and F.
(b) Congruent shapes are: A and D; C and E.

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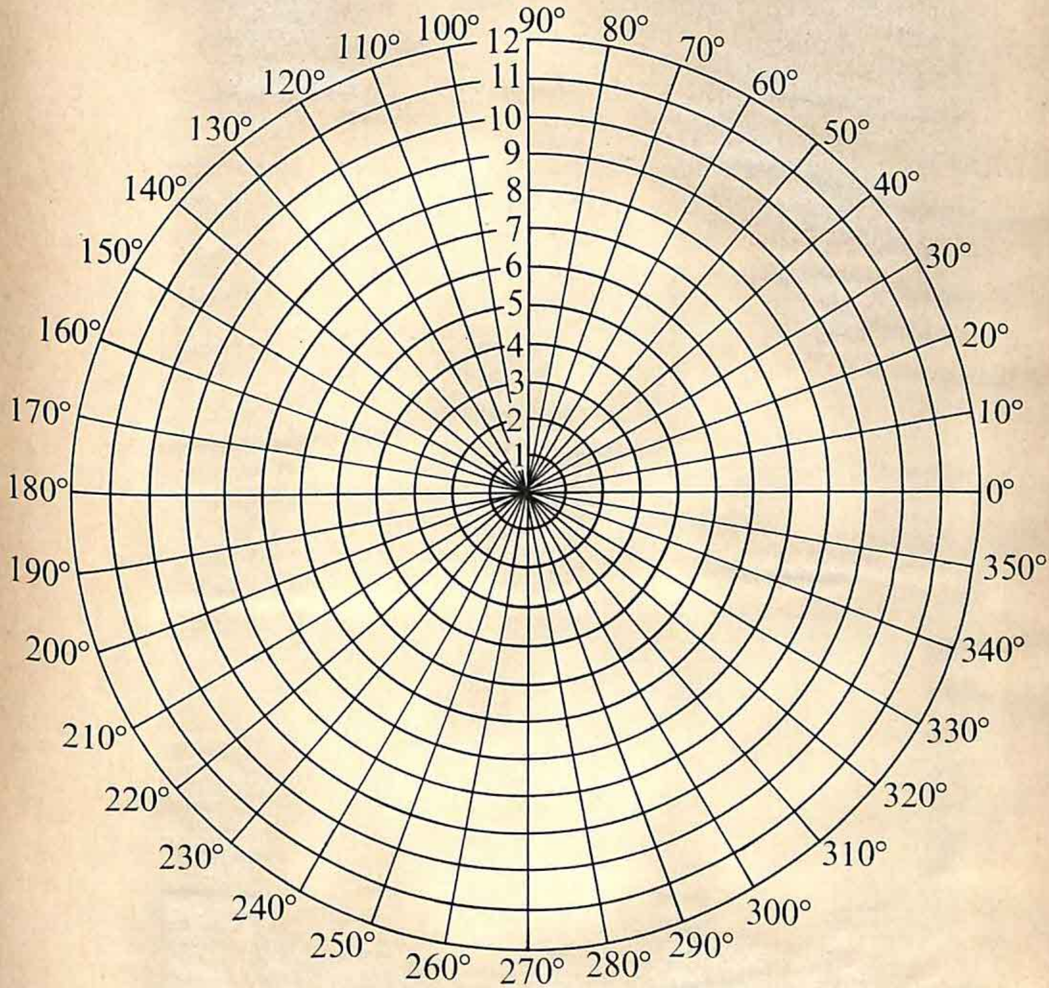

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